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Deviation Inequalities in Infinite Dimensions: an Approach via Covariance Representations

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Abstract

Various deviation inequalities are obtained for functionals on Wiener space, Poisson space or more generally for normal martingales. The method is based on covariance identities obtained via the chaotic representation property, and extends to the binomial process and to the infinite discrete cube $\{-1, 1\}^\infty$.

Key words: Deviation inequalities, covariance identities, chaotic representation property, Clark formula.
Mathematics Subject Classification. 62E20, 60H07, 60G44, 60G57.

1 Introduction

The purpose of the present paper is to further explore topics in deviation inequalities, in particular in infinite dimensional settings. Deviation and concentration have attracted a lot of attention in recent years well summarized in [13, 14] where the reader will find up-to-date information, precise references and credit. Among the various methods used to obtain these results one that we would like to emphasize is based on covariance representations. In particular, it was used in the Gaussian or more generally infinitely divisible cases in [3], [10], [11]. We tackle below the infinite dimensional

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case with a similar method, recovering also the results recently obtained in [2], [4], [27] using (modified) logarithmic Sobolev inequalities.

The content of this paper is as follows. In the next section, we briefly review the notion of normal martingale and recall elements of its structure theory. Applications that are specific to the Gaussian case are mentioned in Section 3. Section 4 is devoted to deviation inequalities for normal martingales having the chaos representation property. This is then specialized to "deterministic" structure equations that simultaneously cover the Poisson and Wiener cases in Section 5. The general case of Poisson random measure on a metric space is treated in Section 6, and the gradient of [5] is used in Section 7 for the Poisson process on $\mathbb{R}_+$. Section 8 is devoted to similar results for the binomial process and on the infinite discrete cube.

2 Preliminaries: normal martingales

Let $(M_t)_{t\in\mathbb{R}_+}$ be a normal martingale, i.e. $(M_t)_{t\in\mathbb{R}_+}$ is a martingale with deterministic angle bracket $d\langle M_t, M_t\rangle = dt$. Let $(\mathcal{F}_t)_{t\in\mathbb{R}_+}$ be the filtration generated by $(M_t)_{t\in\mathbb{R}_+}$ and let $\mathcal{F} = \bigvee_{t\in\mathbb{R}^+} \mathcal{F}_t$. The multiple stochastic integral $I_n(f_n)$ is then defined as

$$I_n(f_n) = n! \int_0^\infty \int_0^{t_n} \cdots \int_0^{t_2} f_n(t_1,\ldots,t_n) dM_{t_1}\cdots dM_{t_n}, \quad f_n \in L^2(\mathbb{R}_+)^{\circ n}, \quad n \geq 1,$$

where $L^2(\mathbb{R}_+)^{on}$ is the set of symmetric square integrable functions on $\mathbb{R}_+^n$, with

$$E[I_n(f_n)I_m(g_m)] = n!\mathbf{1}_{\{n=m\}}\langle f_n, g_m\rangle_{L^2(\mathbb{R}_+)^{\circ n}}. \tag{1}$$

We assume that $(M_t)_{t\in\mathbb{R}_+}$ has the chaos representation property, i.e. every $F \in L^2(\Omega,\mathcal{F},P)$ has a decomposition as $F = \sum_{n=0}^\infty I_n(f_n)$. Let $D : \mathrm{Dom}(D) \longrightarrow L^2(\Omega \times \mathbb{R}_+, dP\times dt)$ denote the closable gradient operator defined as

$$D_tF = \sum_{n=1}^\infty nI_{n-1}(f_n(*,t)), \quad dP\times dt - a.e.,$$

with $F = \sum_{n=0}^\infty I_n(f_n)$. The Clark formula is a consequence of the chaos representation property for $(M_t)_{t\in\mathbb{R}_+}$, and states that any $F \in \mathrm{Dom}(D) \subset L^2(\Omega,\mathcal{F},P)$ has a representation

$$F = E[F] + \int_0^\infty E[D_tF \mid \mathcal{F}_t]dM_t. \tag{2}$$

It admits a simple proof via the chaos expansion of F:

$$\begin{aligned} F &= E[F]+\sum_{n=1}^{\infty} n! \int_0^{\infty} \int_0^{t_n} \cdots \int_0^{t_2} f_n(t_1,\ldots,t_n) dM_{t_1} \cdots dM_{t_n} \\ &= E[F]+\sum_{n=1}^{\infty} n \int_0^{\infty} I_{n-1}(f_n(*,t_n)\mathbf{1}_{\{*<t_n\}}) dM_{t_n} = E[F]+\int_0^{\infty} E[D_tF \mid \mathcal{F}_t] dM_t. \end{aligned}$$

Let $(P_t)_{t\in\mathbb{R}_+}$ denote the Ornstein-Uhlenbeck semi-group, defined as

$$P_tF = \sum_{n=0}^{\infty} e^{-nt} I_n(f_n),$$

with $F = \sum_{n=0}^{\infty} I_n(f_n)$.

Proposition 2.1 *Let $F, G \in \mathrm{Dom}(D)$. We have the covariance identities*

$$\mathrm{Cov}(F,G) = E\left[\int_0^{\infty} D_tF E[D_tG \mid \mathcal{F}_t] dt\right], \tag{3}$$

and

$$\mathrm{Cov}(F,G) = E\left[\int_0^{\infty}\int_0^{\infty} e^{-s} D_uF P_s D_uG du ds\right]. \tag{4}$$

Proof. The first identity is a consequence of the Clark formula. By orthogonality of multiple integrals of different orders and continuity of P_s on $L^2(\Omega)$, it suffices to prove the second identity for $F = I_n(f_n)$ and $G = I_n(g_n)$ as

$$\begin{aligned} E[I_n(f_n)I_n(g_n)] &= n!\langle f_n, g_n\rangle_{L^2(\mathbb{R}_+^n)} = \frac{1}{n} E\left[\int_0^{\infty} D_uF D_uG du\right] \\ &= E\left[\int_0^{\infty} e^{-s} \int_0^{\infty} D_uF P_s D_uG du ds\right]. \end{aligned}$$

□

Relation (4) implies the covariance inequality

$$|\mathrm{Cov}(F,G)| \le \|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} E[\|DG\|_{L^2(\mathbb{R}_+)}]. \tag{5}$$

If $(M_t)_{t\in\mathbb{R}_+}$ is in $L^4(\Omega,\mathcal{F},P)$ then the chaos representation property implies that there exists a square-integrable predictable process $(\phi_t)_{t\in\mathbb{R}_+}$ such that

$$d[M_t, M_t] = dt + \phi_t dM_t, \quad t \in \mathbb{R}_+. \tag{6}$$

This last equation is called a structure equation, cf. [7]. Let $i_t = \mathbf{1}_{\{\phi_t=0\}}$ and $j_t = 1 - i_t = \mathbf{1}_{\{\phi_t\neq 0\}}$, $t \in \mathbb{R}_+$. The continuous part of $(M_t)_{t\in\mathbb{R}_+}$ is given by $dM_t^c = i_t dM_t$ and the eventual jump of $(M_t)_{t\in\mathbb{R}_+}$ at time $t \in \mathbb{R}_+$ is given as $\Delta M_t = \phi_t$ on $\{\Delta M_t \neq 0\}$, $t \in \mathbb{R}_+$, see [7], p. 70. The following are examples of normal martingales with the chaos representation property, cf. [7].

a) $(\phi_t)_{t\in\mathbb{R}_+}$ is deterministic. Then $(M_t)_{t\in\mathbb{R}_+}$ can be represented as

$$dM_t = i_t dB_t + \phi_t(dN_t - \lambda_t dt), \quad t \in \mathbb{R}_+, \quad M_0 = 0, \tag{7}$$

with $\lambda_t = (1 - i_t)1/\phi_t^2$, $t \in \mathbb{R}_+$, where $(B_t)_{t\in\mathbb{R}_+}$ is a standard Brownian motion, and $(N_t)_{t\in\mathbb{R}_+}$ a Poisson process independent of $(B_t)_{t\in\mathbb{R}_+}$, with intensity $\nu_t = \int_0^t \lambda_s ds$, $t \in \mathbb{R}_+$.

b) Azéma martingales where $\phi_t = \beta M_t$, $\beta \in [-2, 0)$.

If $(\phi_t)_{t\in\mathbb{R}_+}$ is a deterministic function, then $i_t D_t$ is still a derivation operator, and we have the product rule

$$D_t(FG) = FD_tG + GD_tF + \phi_t D_tF D_tG, \quad t \in \mathbb{R}_+, \tag{8}$$

cf. Proposition 1.3 of [21]. In fact D_t can be written as

$$D_t = \frac{j_t}{\phi_t}\Delta_t^\phi + i_t D_t, \tag{9}$$

where Δ_t^ϕ is the finite difference operator defined on random functionals by addition at time t of a jump of height ϕ_t to $(M_t)_{t\in\mathbb{R}_+}$. If $\phi_t \neq 0$, this implies

$$D_t e^F = \frac{e^F}{\phi_t}(e^{\phi_t D_t F} - 1), \tag{10}$$

and at the limit $\phi_t \to 0$, D_t becomes a derivation: $D_t e^F = e^F D_t F$.
In the deterministic case, an Ornstein-Uhlenbeck process $(X_t)_{t\in\mathbb{R}_+}$ can be associated to the semi-group $(P_s)_{s\in\mathbb{R}_+}$, and this implies the continuity of P_s.

Lemma 2.1 *Assume that $(\phi_t)_{t\in\mathbb{R}_+}$ is a deterministic function. For $F \in \mathrm{Dom}(D)$ we have*

$$\|P_t DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \leq \|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))}, \quad t \in \mathbb{R}_+. \tag{11}$$

Proof. Let $(M_t)_{t\in\mathbb{R}_+}$ be defined as in (7) on the product space $\Omega = \Omega_1 \times \Omega_2$ of independent Brownian motion $(B_t)_{t\in\mathbb{R}_+}$ and Poisson process $(N_t)_{t\in\mathbb{R}_+}$. The exponential vector

$$\varepsilon(f) = \sum_{n=0}^\infty I_n(f^{\circ n}),$$

$f \in L^1(\mathbb{R}_+) \cap L^2(\mathbb{R}_+)$, has the probabilistic interpretation

$$\varepsilon(f) = \exp\left(\int_0^\infty i_s f(s) dB(s) + \int_0^\infty j_s \log(1 + \phi(s)f(s)) dN(s) \right.$$
$$\left. -\frac{1}{2}\int_0^\infty i(s)f(s)ds - \int_0^\infty j(s)\frac{f(s)}{\phi(s)}ds\right).$$

Let $(X_1^t)_{t\in\mathbb{R}_+}$ and $(X_2^t)_{t\in\mathbb{R}_+}$ be respectively the classical Ornstein-Uhlenbeck process on Wiener space, and the Ornstein-Uhlenbeck process on Poisson space, cf. [24]. We have

$$\begin{aligned}
&E[\varepsilon(f)(X_1^t, X_2^t) \mid (X_1^0, X_2^0)] \\
&= E\left[\exp\left(\int_0^\infty i_s f(s) dX_1^t(s) + \int_0^\infty j_s \log(1+\phi(s)f(s)) dX^t(s)\right.\right. \\
&\quad \left.\left. -\frac{1}{2}\int_0^\infty i(s)f(s)ds - \int_0^\infty j(s)\frac{f(s)}{\phi(s)}ds\right) \mid (X_1(0), X_2(0))\right] \\
&= \exp\left(\int_0^\infty i_s e^{-t} f(s) dX_1^0(s) + \int_0^\infty j_s \log(1+e^{-t}\phi(s)f(s)) dX_2^0(s)\right. \\
&\quad \left. -\frac{1}{2}\int_0^\infty i(s)e^{-t}f(s)ds - \int_0^\infty j(s)e^{-t}\frac{f(s)}{\phi(s)}ds\right). \\
&= \varepsilon(e^{-t}f)(X_1^0, X_2^0) = P_t\varepsilon(f).
\end{aligned}$$

This identity extends to linear combinations of exponential vectors by linearity, and to $L^2(\Omega)$ by density and continuity of P_t. This implies that

$$\|P_t DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \leq \|P_t |DF|_{L^2(\mathbb{R}_+)}\|_{L^\infty(\Omega)} \leq \|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))}, \quad t \in \mathbb{R}_+,$$

for all $F \in \mathrm{Dom}(D)$. $\square$

3 The Gaussian case

Before proceeding to general deviation inequalities for normal martingales with the chaos representation property, we make some remarks on the Gaussian case. If $(M_t)_{t\in\mathbb{R}_+}$ is a standard Brownian motion, then D is a derivation operator whose action on cylindrical functionals of the form

$$F = f(I_1(e_1), \ldots, I_1(e_n)), \quad e_1, \ldots, e_n \in L^2(\mathbb{R}_+), \quad f \in \mathcal{C}_b^1(\mathbb{R}^n), \tag{12}$$

is given by

$$D_t F = \sum_{i=1}^{i=n} e_i(t) \partial_i f(I_1(e_1), \ldots, I_1(e_n)), \quad t \in \mathbb{R}_+,$$

cf. e.g. [25]. We have the relations

$$\|DF\|_{L^2(\mathbb{R}_+)} = |\nabla f|(I_1(e_1), \ldots, I_1(e_n)), \quad a.s.,$$

and

$$\|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} = \|\nabla f\|_{\mathrm{Lip}},$$

where $|\nabla f|$ is the gradient norm of f and $\|\nabla f\|_{\mathrm{Lip}}$ is the Lipschitz norm of f. The covariance inequality (5):

$$|\mathrm{Cov}(F,G)| \leq \|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+)}E[\|DG\|_{L^2(\mathbb{R}_+)}],$$

becomes the extension of Theorem 2.1 of [3] to Wiener functionals $F,G \in \mathrm{Dom}(D)$. If $\{e_1,\ldots,e_n\}$ is orthonormal in $L^2(\mathbb{R}_+)$ then $\{I_1(e_1),\ldots,I_1(e_n)\}$ is a family of independent standard Gaussian random variables, and applying Theorem 2.2 in [3] and Pisier's inequality [18] to F written as in (12) with $\|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))} \leq 1$, we obtain:

$$P(|F-E[F]| \geq x) \leq E[|F-E[F]|]\frac{e^{-x^2/2}}{x} \leq \frac{2}{\sqrt{2\pi}}\frac{e^{-x^2/2}}{x}, \qquad x>0.$$

By density of the cylindrical functionals this result extends to Wiener functionals F in the domain of D and satisfying the condition $\|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))} \leq 1$. We refer to [8] for other deviation inequalities on the Wiener space.

4 Deviation inequality in the general case

In this section we work in the general framework of normal martingales with the chaos representation property.

Lemma 4.1 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$ and $e^{sF} \in \mathrm{Dom}(D)$, $0 < s \leq t_0$ for some $t_0 > 0$. Then*

$$E[e^{t(F-E[F])}] \leq \exp\left(\int_0^t h(s)ds\right), \quad 0 \leq t \leq t_0, \tag{13}$$

where h is defined as

$$h(s) = \int_0^\infty \|D_uF\|_\infty \|e^{-sF}D_ue^{sF}\|_\infty du, \quad s \in [0,t_0]. \tag{14}$$

Proof. Let us first assume that $E[F]=0$. We have

$$\begin{aligned} E[Fe^{sF}] &= E\left[\int_0^\infty E[D_uF \mid \mathcal{F}_u]E[D_ue^{sF} \mid \mathcal{F}_u]du\right] \\ &= E\left[\int_0^\infty D_ue^{sF}E[D_uF \mid \mathcal{F}_u]du\right] \\ &\leq E[e^{sF}]\int_0^\infty \|D_uF\|_\infty \|e^{-sF}D_ue^{sF}\|_\infty du, \quad 0 \leq s \leq t_0. \end{aligned}$$

In the general case, letting $L(s) = E[e^{s(F-E[F])}]$, we have

$$\log(E[e^{t(F-E[F])}]) = \int_0^t \frac{L'(s)}{L(s)}ds \leq \int_0^t \frac{E[(F-E[F])e^{s(F-E[F])}]}{E[e^{s(F-E[F])}]}ds,$$

$0 \leq t \leq t_0$. □

Given $F \in L^2(\Omega)$ we denote by η_F the process

$$\eta_F(t) = E[D_t F \mid \mathcal{F}_t], \quad t \in \mathbb{R}_+,$$

i.e. we have

$$F = E[F] + \int_0^\infty \eta_F(t) dM_t.$$

A modification of the above proof as

$$\begin{aligned} E[Fe^{sF}] &= E\left[\int_0^\infty D_u e^{sF} \eta_u du\right] \leq E\left[e^{sF} \|e^{-sF} De^{sF}\|_{L^2(\mathbb{R}_+)} \|\eta\|_{L^2(\mathbb{R}_+)}\right] \\ &\leq E\left[e^{sF}\right] \|e^{-sF} De^{sF}\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \|\eta\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))}, \end{aligned}$$

also yields (13), with

$$h(s) = \|\eta\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \|e^{-sF} De^{sF}\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))}.$$

In the next lemma we apply the semi-group correlation identity (4), and refer to [15] for other applications of semi-groups, in particular to logarithmic Sobolev inequalities. This result holds in particular in case of a deterministic structure equation.

Lemma 4.2 *Let us assume that $(P_t)_{t \in \mathbb{R}_+}$ satisfies (11). Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0 |F|}] < \infty$ and $e^{sF} \in \mathrm{Dom}(D)$, $0 < s \leq t_0$, for some $t_0 > 0$. Then*

$$E[e^{t(F - E[F])}] \leq \exp\left(\int_0^t h(s) ds\right), \quad 0 \leq t \leq t_0, \tag{15}$$

where h is any of the functions

$$h(s) = \|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \|e^{-sF} De^{sF}\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))}, \quad s \in [0, t_0], \tag{16}$$

$$h(s) = \left\| \frac{e^{-sF} De^{sF}}{D_u F} \right\|_\infty \|D_u F\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}, \quad s \in [0, t_0]. \tag{17}$$

Proof. Let us first assume that $E[F] = 0$. If the Ornstein-Uhlenbeck semi-group satisfies (11), then

$$\begin{aligned} E[Fe^{sF}] &= E\left[\int_0^\infty e^{-v} \int_0^\infty D_u e^{sF} P_v D_u F du dv\right] \\ &\leq E\left[e^{sF} \|e^{-sF} De^{sF}\|_{L^2(\mathbb{R}_+)} \int_0^\infty e^{-v} \|P_v DF\|_{L^2(\mathbb{R}_+)} dv\right] \\ &\leq E\left[e^{sF}\right] \|e^{-sF} De^{sF}\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \left\| \int_0^\infty e^{-v} P_v \|DF\|_{L^2(\mathbb{R}_+)} dv \right\|_\infty \\ &\leq E\left[e^{sF}\right] \|e^{-sF} De^{sF}\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \int_0^\infty e^{-v} \|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} dv \\ &\leq E\left[e^{sF}\right] \|e^{-sF} De^{sF}\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))}. \end{aligned}$$

A similar argument shows that

$$\begin{aligned}
E[Fe^{sF}] &= E\left[\int_0^\infty e^{-v}\int_0^\infty D_u e^{sF} P_v D_u F du dv\right] \\
&\leq E\left[e^{sF}\left\|\frac{e^{-sF}De^{sF}}{DF}\right\|_\infty \int_0^\infty e^{-v}\|DFP_vDF\|_{L^1(\mathbb{R}_+)}dv\right] \\
&\leq E\left[e^{sF}\left\|\frac{e^{-sF}De^{sF}}{DF}\right\|_\infty \int_0^\infty e^{-v}\|DF\|_{L^2(\mathbb{R}_+)}\|P_vDF\|_{L^2(\mathbb{R}_+)}dv\right] \\
&\leq E\left[e^{sF}\right]\left\|\frac{e^{-sF}De^{sF}}{DF}\right\|_\infty \|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}\left\|\int_0^\infty e^{-v}P_v\|DF\|_{L^2(\mathbb{R}_+)}dv\right\|_\infty \\
&\leq E\left[e^{sF}\right]\left\|\frac{e^{-sF}De^{sF}}{DF}\right\|_\infty \|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}\int_0^\infty e^{-v}\|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}dv \\
&\leq E\left[e^{sF}\right]\left\|\frac{e^{-sF}De^{sF}}{DF}\right\|_\infty \|DF\|^2_{L^\infty(\Omega,L^2(\mathbb{R}_+))}.
\end{aligned}$$

The remainder of the proof is as in Lemma 4.1. □

From these lemmas we obtain a general deviation inequality.

Proposition 4.1 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$ and $e^{sF} \in \mathrm{Dom}(D)$, $0 < s \leq t_0$, for some $t_0 > 0$. Let h be the function defined in* (14)*, or (if $(\phi_t)_{t\in\mathbb{R}_+}$ is deterministic) in* (16) *or* (17)*. Then*

$$P(F - E[F] \geq x) \leq \exp\left(-\int_0^x h^{-1}(s)ds\right), \quad 0 < x < h(t_0),$$

where h^{-1} is the inverse of h.

Proof. From Lemma 4.1 we have for all $x \in \mathbb{R}_+$:

$$e^{tx}P(F - E[F] \geq x) \leq E[e^{t(F-E[F])}] \leq e^{H(t)}, \quad 0 \leq t \leq t_0,$$

with

$$H(t) = \int_0^t h(s)ds, \quad 0 \leq t \leq t_0.$$

For any $0 < t < t_0$ we have $\frac{d}{dt}(H(t) - tx) = h(t) - x$, hence

$$\begin{aligned}
\min_{0<t<t_0}(H(t) - tx) &= H(h^{-1}(x)) - xh^{-1}(x) = \int_0^{h^{-1}(x)} h(s)ds - xh^{-1}(x) \\
&= \int_0^x s dh^{-1}(s) - xh^{-1}(x) = -\int_0^x h^{-1}(s)ds.
\end{aligned}$$

□

5 Deviation inequality for deterministic structure

In this section we work with $(\phi_t)_{t\in\mathbb{R}_+}$ a deterministic function, i.e. $(M_t)_{t\in\mathbb{R}_+}$ is written as in (7). This covers the Gaussian case for $\phi = 0$, and also the general Poisson case, as shown in Sect. 6.

Theorem 5.1 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$. Then*

$$P(F - E[F] \geq x) \leq \exp\left(-\int_0^x h^{-1}(s)ds\right), \quad 0 < x < h(t_0),$$

where h^{-1} is the inverse of any of the following functions:

$$h(t) = \int_0^\infty \frac{j_u}{|\phi_u|}\|D_uF\|_\infty(e^{t|\phi_u|\|D_uF\|_\infty} - 1)du + t\int_0^\infty i_u\|D_uF\|_\infty^2 du, \tag{18}$$

$$h(t) = \|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}\|\phi^{-1}(e^{t|\phi DF|} - 1)\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}, \tag{19}$$

$$h(t) = \left\|\frac{1}{\phi DF}(e^{t\phi DF} - 1)\right\|_\infty \|D_uF\|^2_{L^\infty(\Omega,L^2(\mathbb{R}_+))}, \quad t \in [0, t_0]. \tag{20}$$

Proof. In the deterministic case, $e^{-tF}D_ue^{tF} \in L^2(\Omega \times \mathbb{R}_+)$, with

$$e^{-tF}D_ue^{tF} = \frac{j_u}{\phi_u}\left(e^{t\phi_uD_uF} - 1\right) + i_utD_uF, \quad u \in \mathbb{R}_+, \tag{21}$$

which can also be written as

$$e^{-tF}D_ue^{tF} = \frac{1}{\phi_u}\left(e^{t\phi_uD_uF} - 1\right), \tag{22}$$

by replacing $\phi_u^{-1}\left(e^{t\phi_uD_uF} - 1\right)$ with its limit as $\phi_u \to 0$, i.e. tD_uF, if $\phi_u = 0$. It remains to apply Proposition 4.1. □

Note that the inequalities given by (18), (19) and (20) are not comparable. Using the bound

$$|\phi_u^{-1}(e^{t\phi_uD_uF} - 1)| \leq t|D_uF|e^{t|\phi_uD_uF|},$$

for all values of $\phi_u \in \mathbb{R}$, Theorem 5.1 also holds for the functions

$$h(t) = t\int_0^\infty \|D_uF\|_\infty^2\|e^{t|\phi_uD_uF|}\|_\infty du,$$

and

$$h(t) = t\|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}\|e^{t|\phi DF|}DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))}, \quad t \in [0, t_0].$$

Results of the type presented above continue to hold if for all $0 \leq t \leq t_0$ and $E[F] = 0$, $E[Fe^{tF}] \leq h(t)E[e^{tF}]$ for some increasing non negative continuous function h such that $h(0) = 0$. We will show in the rest of the paper many instances where we can estimate h and h^{-1}.

Theorem 5.2 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$, and $\phi_u D_u F \le K(u)$ a.s., $u \in \mathbb{R}_+$ for some function $K : \mathbb{R}_+ \to \mathbb{R}$. Then*

$$P(F - E[F] \ge x) \le \exp\left(- \int_0^x h^{-1}(s) ds \right), \quad 0 < x < h(t_0),$$

where h^{-1} is the inverse of

$$h(t) = \left\| \frac{1}{K(\cdot)} (e^{tK(\cdot)} - 1) \right\|_\infty \|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}, \quad t \in [0, t_0].$$

Proof. Since the function $x \mapsto (e^x - 1)/x$ is positive and increasing on $\mathbb{R}$, we have

$$0 \le \frac{e^{-tF} D_u e^{tF}}{D_u F} = \frac{1}{\phi_u D_u F} \left(e^{t\phi_u D_u F} - 1 \right) \le \frac{1}{K(u)} (e^{tK(u)} - 1), \quad u \in \mathbb{R}_+,$$

and

$$\left| \frac{e^{-tF} D_u e^{tF}}{D_u F} \right| \le \frac{1}{K(u)} (e^{tK(u)} - 1), \quad u \in \mathbb{R}_+.$$

It remains to apply Proposition 4.1 and Lemma 4.2. □

In each of the following deviation inequalities the interval of validity for $x \in \mathbb{R}_+$ can be easily determined in terms of the function h, and will not be explicitly given. The following corollary is the main result of this section.

Corollary 5.1 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$, $\phi DF \le K$ for some $K \ge 0$ and $\|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} < \infty$. Then*

$$\begin{aligned} P(F - E[F] \ge x) &\le \exp\left(- \frac{\|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}}{K^2} g\left(\frac{xK}{\|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}} \right) \right) \\ &\le \exp\left(- \frac{x}{2K} \log\left(1 + \frac{xK}{\|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}} \right) \right), \end{aligned}$$

with $g(u) = (1+u)\log(1+u) - u$, $u \ge 0$. If $K = 0$ (decreasing functionals) we have

$$P(F - E[F] \ge x) \le \exp\left(- \frac{x^2}{2\|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}} \right). \tag{23}$$

Proof. The function h defined in Theorem 5.2 satisfies

$$h(t) \le \frac{1}{K} (e^{tK} - 1) \|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))},$$

hence

$$\begin{aligned} - \int_0^x h^{-1}(t) dt &\le - \frac{1}{K} \int_0^x \log\left(1 + tK \|DF\|^{-2}_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \right) dt \\ &= - \frac{1}{K} \left((x + \frac{1}{K} \|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}) \log\left(1 + xK \|DF\|^{-2}_{L^\infty(\Omega, L^2(\mathbb{R}_+))} \right) - x \right). \end{aligned}$$

If $K = 0$, the above proof is still valid by replacing all terms by their limits as $K \to 0$.

□

For constant $\phi_t = \phi \in \mathbb{R}_+$, $t \in \mathbb{R}_+$, we have the following.

Corollary 5.2 *Assume that* $\phi_t = \phi \in \mathbb{R}_+$, $t \in \mathbb{R}_+$, *is constant. Let* $F \in \mathrm{Dom}(D)$ *be such that* $E[e^{t_0|F|}] < \infty$, $DF \leq K$ *for some* $K \geq 0$ *and* $\|DF\|_{L^\infty(\Omega, L^2(\mathbb{R}_+))} < \infty$. *Then*

$$\begin{aligned} P(F - E[F] \geq x) &\leq \exp\left(-\frac{\|DF\|^2_{L^\infty(\Omega,L^2(\mathbb{R}_+))}}{\phi^2 K^2} g\left(\frac{x\phi K}{\|DF\|^2_{L^\infty(\Omega,L^2(\mathbb{R}_+))}}\right)\right) \\ &\leq \exp\left(-\frac{x}{2\phi K}\log\left(1 + \frac{x\phi K}{\|DF\|^2_{L^\infty(\Omega,L^2(\mathbb{R}_+))}}\right)\right), \end{aligned}$$

with $g(u) = (1+u)\log(1+u) - u$, $u \geq 0$. *If* $\phi = 0$ *(Wiener case) or* $K = 0$ *(decreasing functionals) we have*

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2\|DF\|^2_{L^\infty(\Omega,L^2(\mathbb{R}_+))}}\right). \tag{24}$$

Proof. We apply Corollary 5.1 with the condition $\phi DF \leq \phi K$. □

In particular if F is $\mathcal{F}_T$-measurable, then $\|DF\|_{L^\infty(\Omega,L^2(\mathbb{R}_+))} \leq KT$ and

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{T}{\phi^2} g\left(\frac{\phi x}{KT}\right)\right) \leq \exp\left(-\frac{x}{2K}\log\left(1 + \frac{\phi x}{KT}\right)\right),$$

which can be compared to the inequality

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x}{4K}\log\left(1 + \frac{x}{2TK^2}\right)\right). \tag{25}$$

which follows from Proposition 6.1 in [2]. We refer to [18], [26], for the classical inequality (23) in the case $\phi = 0$, i.e. on Wiener space, which gives

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2K^2 T}\right).$$

Corollary 5.3 *Let* $\phi_t = \phi \in \mathbb{R}_+$, $t \in \mathbb{R}_+$, *be constant. Let* $F \in \mathrm{Dom}(D)$ *be such that* $E[e^{t_0|F|}] < \infty$, $\|DF\|_\infty \leq K$ *and* $\|DF\|_{L^1(\mathbb{R}_+, L^\infty(\Omega))} < \infty$. *Then*

$$\begin{aligned} P(F - E[F] \geq x) &\leq \exp\left(-\frac{\|DF\|_{L^1(\mathbb{R}_+,L^\infty(\Omega))}}{\phi^2 K} g\left(\frac{x\phi}{\|DF\|_{L^1(\mathbb{R}_+,L^\infty(\Omega))}}\right)\right) \\ &\leq \exp\left(-\frac{x}{2\phi K}\log\left(1 + \frac{x\phi}{\|DF\|_{L^1(\mathbb{R}_+,L^\infty(\Omega))}}\right)\right), \end{aligned}$$

with $g(u) = (1+u)\log(1+u) - u$, $u \geq 0$.

Proof. The function defined in (18) of Theorem 5.1 satisfies

$$h(t) \leq \phi^{-1}(e^{t\phi K} - 1)\|DF\|_{L^1(\mathbb{R}_+,L^\infty(\Omega))},$$

which allows to follow the proof of Corollary 5.1. □

The proof of Corollary 5.3 applies also in the limiting case $\phi = 0$, and gives the following corollary.

Corollary 5.4 *Let $\phi_t = 0$, $t \in \mathbb{R}_+$. Let $F \in \mathrm{Dom}(D)$ be such that $\|DF\|_\infty \leq K$ and $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$. Then*

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2K\|DF\|_{L^1(\mathbb{R}_+, L^\infty(\Omega))}}\right).$$

Proof. We have $h(t) = tK\|DF\|_{L^1(\mathbb{R}_+, L^\infty(\Omega))}$, hence $-h^{-1}(t) = -tK^{-1}\|DF\|^{-1}_{L^1(\mathbb{R}_+, L^\infty(\Omega))}$. $\square$

Corollaries 5.3 and 5.4 are weaker than Corollary 5.2, however they rely only on (18) or on Lemma 4.1, and not on the use of $(P_t)_{t\in\mathbb{R}_+}$. For this reason, Corollary 5.4 can be stated for any derivation operator D that can be used to write a Clark formula, and in particular it transfers immediately to the Poisson space for the operator $\tilde{D}$, see Sect. 7.

6 Difference Operator on Poisson space

Ordering of configurations is not important for the statement of the results of Sect. 5 on the Poisson space, hence in this case there is no reason to restrict the index set to $\mathbb{R}_+$. Let X be a Polish space and let Ω^X denote the set of Radon measures

$$\Omega^X = \left\{ \omega = \sum_{i=1}^{i=N} \epsilon_{t_i} \ : \ (t_i)_{i=1}^{i=N} \subset X, \ t_i \neq t_j, \ \forall i \neq j, \ N \in \mathbb{N} \cup \{\infty\} \right\},$$

where ϵ_t denotes the Dirac measure at $t \in X$. Given $A \in \mathcal{B}(X)$, let $\mathcal{F}_A = \sigma(\omega(B) \ : \ B \in \mathcal{B}(X), \ B \subset A)$. Let σ be a diffuse Radon measure on X, and let P denote the Poisson measure with intensity σ on Ω^X and let $L^2_\sigma(X) = L^2(X, \sigma)$. The multiple integral $I_n(f_n)$ is defined as

$$I_n(f_n)(\omega) = \int_{\Delta_n} f_n(t_1, \ldots, t_n)(\omega(dt_1) - \sigma(dt_1)) \cdots (\omega(dt_n) - \sigma(dt_n)), \quad f_n \in L^2_\sigma(X^n)^{\circ n},$$

with $\Delta_n = \{(t_1, \ldots, t_n) \in X^n \ : \ t_i \neq t_j, \ \forall i \neq j\}$, and the isometry formula

$$E[I_n(f_n) I_m(g_m)] = n! \mathbf{1}_{\{n=m\}} \langle f_n, g_m \rangle_{L^2_\sigma(X)^{\circ n}},$$

see [17]. Moreover every square-integrable random variable $F \in L^2(\Omega^X, P)$ admits the Wiener-Poisson decomposition

$$F = \sum_{n=0}^{\infty} I_n(f_n)$$

in series of multiple stochastic integrals. The linear closable operator

$$D : L^2(\Omega^X, P) \to L^2(\Omega^X \times X, P \otimes \sigma)$$

is defined as

$$D_t I_n(f_n)(\omega) = n I_{n-1}(f_n(*, t))(\omega), \quad P(d\omega) \otimes \sigma(dt) - a.e., \quad n \in \mathbb{N}.$$

It is known, cf. [12] or Proposition 1 of [17], that

$$D_t F(\omega) = F(\omega \cup \{t\}) - F(\omega), \qquad dP \times dt - a.e., \quad F \in \mathrm{Dom}(D),$$

where as a convention we identify $\omega \in \Omega^X$ with its support. There exists a measurable bijection $\tau : X \to \mathbb{R}_+$ such that the Lebesgue measure is the image of σ by τ, and this allows to restate Corollary 5.1 and Corollary 5.3. In this way we obtain a different proof of Proposition 3.1. in [27]:

Corollary 6.1 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$, $DF \leq K$, a.s., for some $K \geq 0$, and $\|DF\|_{L^\infty(\Omega, L^2(X))} < \infty$. Then*

$$\begin{aligned} P(F - E[F] \geq x) &\leq \exp\left(-\frac{\|DF\|^2_{L^\infty(\Omega,L^2(X))}}{K^2} g\left(\frac{xK}{\|DF\|^2_{L^\infty(\Omega,L^2(X))}}\right)\right) \\ &\leq \exp\left(-\frac{x}{2K}\log\left(1 + \frac{xK}{\|DF\|^2_{L^\infty(\Omega,L^2(X))}}\right)\right), \end{aligned}$$

with $g(u) = (1+u)\log(1+u) - u$, $u \geq 0$. If $K = 0$ (decreasing functionals) we have

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2\|DF\|^2_{L^\infty(\Omega,L^2(X))}}\right). \tag{26}$$

In particular if $F = \int_X f(x)\omega(dx)$ we have $\|DF\|_{L^\infty(\Omega,L^2(X))} = \|f\|_{L^2(X)}$ and if $f \leq K$, a.s., then

$$P\left(\int_X f(x)(\omega(dx) - \sigma(dx)) \geq x\right) \leq \exp\left(-\frac{\int_X |f^2(x)|\sigma(dx)}{K^2} g\left(\frac{xK}{\int_X |f^2(x)|\sigma(dx)}\right)\right),$$

which covers Proposition 2 of [23]. If $f \leq 0$, a.s., then

$$P\left(\int_X f(x)(\omega(dx) - \sigma(dx)) \geq x\right) \leq \exp\left(-\frac{x^2}{2\int_X |f^2(x)|\sigma(dx)}\right).$$

Corollary 6.1 also implies, for $F \in \mathrm{Dom}(D)$ such that $E[e^{t_0|F|}] < \infty$ and $\|DF\|_\infty \leq K$:

$$\begin{aligned} P(F - E[F] \geq x) &\leq \exp\left(-\frac{\|DF\|_{L^1(X,L^\infty(\Omega))}}{K} g\left(\frac{x}{\|DF\|_{L^1(X,L^\infty(\Omega))}}\right)\right) \\ &\leq \exp\left(-\frac{x}{2K}\log\left(1 + \frac{x}{\|DF\|_{L^1(X,L^\infty(\Omega))}}\right)\right). \end{aligned}$$

If $F = \int_X f(x)\omega(dx)$ then $\|DF\|_{L^1(X,L^\infty(\Omega))} = \|f\|_{L^1(X)}$, and if moreover $\|f\|_\infty \leq K$ we obtain

$$P\left(\int_X f(x)(\omega(dx) - \sigma(dx)) \geq x\right) \leq \exp\left(-\frac{\int_X |f(x)|\sigma(dx)}{K} g\left(\frac{x}{\int_X |f(x)|\sigma(dx)}\right)\right).$$

In case $f \geq 0$ a.s., this can be written as

$$P\left(\int_X f(x)(\omega(dx) - \sigma(dx)) \geq x\right) \leq \exp\left(-\frac{E[F]}{K} g\left(\frac{x}{E[F]}\right)\right).$$

If F is $\mathcal{F}_A$-measurable with $\sigma(A) < \infty$ and $\|DF\|_\infty \leq K$, then $\|DF\|_{L^1(X,L^\infty(\Omega))} \leq K\sigma(A)$ and

$$P(F - E[F] \geq x) \leq \exp\left(-\sigma(A) g\left(\frac{x}{K\sigma(A)}\right)\right) \leq \exp\left(-\frac{x}{2K}\log\left(1 + \frac{x}{K\sigma(A)}\right)\right).$$

As an application we consider as in [23] a family $(\Psi_a)_{a\in\mathbb{N}} \subset L^2(X)$ of functions valued in $[0, K]$ with $\sigma(X) < \infty$, and the functional

$$F = \sup_{a\in\mathbb{N}} \int_X \Psi_a(x)\omega(dx).$$

We have

$$0 \leq D_x F = \sup_{a\in\mathbb{N}} \left(\int_X \Psi_a(x)\omega(dx) + \Psi_a(x)\right) - \sup_{a\in\mathbb{N}} \int_X \Psi_a(x)\omega(dx),$$

hence

$$0 \leq D_x F \leq \sup_{a\in\mathbb{N}} \Psi_a(x) \leq K,$$

and

$$P(F - E[F] \geq x) \leq \exp\left(-\sigma(X) g\left(\frac{x}{K\sigma(X)}\right)\right).$$

In particular for $\omega = \{x\}$ we have

$$D_x F = \sup_{a\in\mathbb{N}}(\Psi_a(x) + \Psi_a(x)) - \sup_{a\in\mathbb{N}} \Psi_a(x) = \sup_{a\in\mathbb{N}} \Psi_a(x),$$

hence

$$\|D_x F\|_\infty \geq \sup_{a\in\mathbb{N}} \Psi_a(x),$$

i.e.

$$\|D_x F\|_\infty = \sup_{a\in\mathbb{N}} \Psi_a(x), \qquad x \in X,$$

and

$$
\begin{aligned}
\|DF\|_{L^1(X,L^\infty(\Omega))} &= \int_X \left(\sup_{a\in\mathbb{N}} \Psi_a(x) \right) \sigma(dx) \\
&= E\left[\int_X \left(\sup_{a\in\mathbb{N}} \Psi_a(x) \right) \omega(dx) \right] \\
&\geq E\left[\sup_{a\in\mathbb{N}} \int_X \Psi_a(x)\omega(dx) \right] = E[F] \\
&= \sum_{n=1}^\infty \frac{e^{-\sigma(X)}}{n!} \int_{X^n} \sup_{a\in\mathbb{N}} (\Psi_a(x_1) + \cdots + \Psi_a(x_n))\sigma(dx_1)\cdots\sigma(dx_n) \\
&\geq \sum_{n=1}^\infty \frac{e^{-\sigma(X)}}{n!} \int_{X^n} \sup_{a\in\mathbb{N}} \Psi_a(x_1)\sigma(dx_1)\cdots\sigma(dx_n) \\
&\geq \sum_{n=1}^\infty \frac{e^{-\sigma(X)}}{n!} \int_{X^n} \|D_{x_1}F\|_\infty \sigma(dx_1)\cdots\sigma(dx_n) \\
&\geq \|DF\|_{L^1(X,L^\infty(\Omega))} \sum_{n=1}^\infty \frac{e^{-\sigma(X)}}{n!} (\sigma(X))^{n-1} \\
&\geq \frac{1}{\sigma(X)} \|DF\|_{L^1(X,L^\infty(\Omega))} (1 - e^{-\sigma(X)}),
\end{aligned}
$$

hence

$$
E[F] \leq \|DF\|_{L^1(X,L^\infty(\Omega))} \leq \frac{\sigma(X)}{1 - e^{-\sigma(X)}} E[F],
$$

and

$$
P(F - E[F] \geq x) \leq \exp\left(-\frac{\sigma(X)}{K(1 - e^{-\sigma(X)})} E[F] g\left(\frac{x(1 - e^{-\sigma(X)})}{\sigma(X)E[F]} \right) \right).
$$

A similar result will hold for

$$
F = f\left(\sup_{a\in\mathbb{N}} \int_X \Psi_a(x)\omega(dx) \right)
$$

with f a Lipschitz function.

7 Local gradient on Poisson space

In the Poisson case, if $X = \mathbb{R}_+$ and σ is the Lebesgue measure, then a local gradient can be introduced, cf. [5], [6], [19]. Let $(T_k)_{k\geq 1}$ denote the jump times of the canonical Poisson process $(N_t)_{t\in\mathbb{R}_+}$, and let $\tau_k = T_k - T_{k-1}$, $k \geq 1$, denote its interjump times, with $T_0 = 0$. Let $\mathcal{S}$ denote the set of smooth random functionals F of the form

$$
F = f(\tau_1, \ldots, \tau_n), \quad f \in \mathcal{C}_c^1(\mathbb{R}_+^n), \quad n \geq 1.
$$

Let $\tilde{D}$ denote the closable gradient defined as

$$\tilde{D}_t F = -\sum_{k=1}^{k=n} \mathbf{1}_{[T_k, T_{k+1}[}(t) \partial_k f(\tau_1, \ldots, \tau_n), \quad t \in \mathbb{R}_+, \quad F \in \mathcal{S}.$$

We have the relation $E[D_t F \mid \mathcal{F}_t] = E[\tilde{D}_t F \mid \mathcal{F}_t]$, $t \in \mathbb{R}_+$, and the Clark formula can be written for $F \in \mathrm{Dom}(\tilde{D})$ as:

$$F = E[F] + \int_0^\infty E[\tilde{D}_t F \mid \mathcal{F}_t] d(N_t - t), \tag{27}$$

cf. Theorem 1 of [19]. First we note that the Wiener space proof of Corollary 5.4 is valid on Poisson space for $\tilde{D}$ which satisfies the chain rule of derivation and Clark formula (27):

Corollary 7.1 *Let $F \in \mathrm{Dom}(\tilde{D})$ be such that $\|DF\|_\infty \leq K$ and $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$. Then*

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2K\|\tilde{D}F\|_{L^1(\mathbb{R}_+, L^\infty(\Omega))}} \right).$$

In particular if F is $\mathcal{F}_T$ measurable and $\|\tilde{D}F\|_\infty \leq K$ then

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2K^2 T} \right), \quad x \geq 0.$$

We construct the exponential random variables $(\tau_k)_{k\geq 1}$ as half sums of squared independent Gaussian random variables. Let $F = f(\tau_1, \ldots, \tau_n)$, and consider the Wiener functional ΘF given as

$$\Theta F = f\left(\frac{x_1^2 + y_1^2}{2}, \ldots, \frac{x_n^2 + y_n^2}{2} \right),$$

where $x_1, \ldots, x_n$, $y_1, \ldots, y_n$, denote two independent collections of normal random variables that may be constructed as Brownian single stochastic integrals. We use the fact that F and ΘF have same law, with the relation

$$2|\tilde{D}F|^2_{L^2(\mathbb{R}_+)} = |\hat{D}\Theta F|^2_{L^2(\mathbb{R}_+)}, \tag{28}$$

see Lemma 1 of [20].

Corollary 7.2 *Let $F \in \mathrm{Dom}(\tilde{D})$ be such that $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$. Then*

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{4\|DF\|^2_{L^\infty(\Omega, L^2(\mathbb{R}_+))}} \right).$$

Proof. We apply the Wiener space counterpart of this result (Corollary 5.2) to ΘF and use Relation (28). □

Note that Corollaries 7.1 and 7.2 are not comparable, unlike Corollaries 5.2 and 5.3. The above result can also be obtained in terms of logarithmic Sobolev inequalities, i.e. by application of Corollary 2.5 of [14] to Theorem 0.7 in [1] (or Relation (4.4) in [14] for a formulation in terms of exponential random variables). A sufficient condition for the exponential integrability of F is $\| |\tilde{D}F|_{L^2(\mathbb{R}_+)} \|_\infty < \infty$, cf. Theorem 4 of [20]. As an example we may consider $F = f(\tau_1, \dots, \tau_n)$ with

$$\sum_{k=1}^{k=n} \tau_k (\partial_k f(\tau_1, \dots, \tau_n))^2 \leq K^2, \quad a.s.$$

8 Discrete settings

The covariance representations (3) and (4) which lead (when applied to F and e^{tF}) to the deviation inequalities of the previous sections has versions in discrete settings. It is thus now our purpose to explore deviation consequences of this representation. We consider the discrete structure equation

$$Y_k^2 = 1 + \varphi_k Y_k, \quad k \in \mathbb{N}, \tag{29}$$

i.e. $(\varphi_k)_{k\in\mathbb{N}}$ is a deterministic sequence of real numbers, and $(Y_k)_{k\geq 1}$ is a sequence of centered independent random variables. Since (29) is a second order equation, there is a family $(X_k)_{k\geq 1}$ of independent Bernoulli $\{-1,1\}$-valued random variables such that

$$Y_k = \frac{\varphi_k + X_k\sqrt{\varphi_k^2+4}}{2}, \quad k \geq 1.$$

The family $(X_k)_{k\in\mathbb{N}}$ is constructed as a family of canonical projections on $\Omega = \{-1,1\}^{\mathbb{N}}$, under the measure P determined from the condition (29) and the fact that $E[Y_k] = 0$ (which imply that $E[Y_k^2] = 1$), i.e.

$$p_k = P(X_k = 1) = P\left(Y_k = \sqrt{\frac{q_k}{p_k}}\right) = \frac{1}{2} - \frac{\varphi_k}{2\sqrt{\varphi_k^2 + 4}}, \quad k \in \mathbb{N},$$

and

$$q_k = P(X_k = -1) = P\left(Y_k = -\sqrt{\frac{p_k}{q_k}}\right) = \frac{1}{2} + \frac{\varphi_k}{2\sqrt{\varphi_k^2 + 4}} \quad k \in \mathbb{N}.$$

Let $J_n(f_n)$ denote the multiple stochastic integral of $f_n \in l^2(\mathbb{N})^{\circ n}$ (the space of square-summable symmetric functions on $\mathbb{N}^n$), defined as

$$\begin{aligned} J_n(f_n) &= \sum_{(k_1,\dots,k_n)\in\Delta_n} f_n(k_1,\dots,k_n) Y_{k_1}\cdots Y_{k_n} \\ &= n! \sum_{k_n=0}^{\infty} \sum_{0\leq k_{n-1}<k_n} \cdots \sum_{0\leq k_1<k_2} f_n(k_1,\dots,k_n) Y_{k_1}\cdots Y_{k_n}, \end{aligned}$$

where

$$\Delta_n = \{(k_1, \ldots, k_n) \in \mathbb{N}^n \ : \ k_i \neq k_j, \ 1 \leq i < j \leq n\},$$

with the isometry

$$E[J_n(f_n)J_m(g_m)] = n!\mathbf{1}_{\{n=m\}}\langle \mathbf{1}_{\Delta_n} f_n, g_m\rangle_{l^2(\mathbb{N})^{\otimes n}}.$$

Let $S_n = \sum_{k=0}^{k=n}(X_k+1)/2$ be the random walk associated to $(X_k)_{k\geq 0}$, cf. also [9], [16]. If $p_k = p$ and $q_k = q$, $k \in \mathbb{N}$, then $J_n(\mathbf{1}_{[0,N]})$ is the Krawtchouk polynomial $K_n(S_N; N+1, p)$ of order n, with parameter $(N+1, p)$, cf. [22]. The set $\mathcal{P}$ of polynomials in $X_1, X_2, X_3, \ldots$ is dense in $L^2(\Omega, P)$, hence any $F \in L^2(\Omega, P)$ can be represented as a series of multiple stochastic integrals:

$$F = \sum_{n=0}^{\infty} J_n(f_n), \quad f_k \in l^2(\mathbb{N})^{\circ k}, \ k \geq 0, \quad J_0(f_0) = E[F].$$

Definition 8.1 *We densely define the linear gradient operator* $D : L^2(\Omega) \longrightarrow L^2(\Omega \times \mathbb{N})$ *as*

$$D_k J_n(f_n) = nJ_{n-1}(f_n(*, k)\mathbf{1}_{\Delta_n}(*, k)), \qquad f_n \in l^2(\mathbb{N})^{\circ n}, \quad n \in \mathbb{N}.$$

We have for $(k_1, \ldots, k_n) \in \Delta_n$

$$D_k\left(\prod_{i=1}^{i=n} Y_{k_i}\right) = \mathbf{1}_{\{l\in\{k_1,\ldots,k_n\}\}} \prod_{\substack{i=1 \\ k_i\neq k}}^{i=n} Y_{k_i},$$

hence the probabilistic interpretation of D_k is

$$D_k F(S.) = \frac{1}{\sqrt{p_k q_k}}\left(F(S. + \mathbf{1}_{\{X_k=-1\}}\mathbf{1}_{\{k\leq \cdot\}}) - F(S. - \mathbf{1}_{\{X_k=1\}}\mathbf{1}_{\{k\leq \cdot\}})\right).$$

In the symmetric case ($p_k = q_k = 1/2$, $k \in \mathbb{N}$), when restricted to cylindrical functionals of the form

$$F = f(X_1, \ldots, X_n),$$

the gradient D is exactly the finite difference operator considered in [3]:

$$D_k F = \frac{1}{\sqrt{p_k q_k}}\left(f(X_1, \ldots, X_{k-1}, +1, X_{k+1}, \ldots, X_n) - f(X_1, \ldots, X_{k-1}, -1, X_{k+1}, \ldots, X_n)\right).$$

The operator D does not satisfy the same product rules as in the continuous time case (Relation (8)).

Proposition 8.1 *Let $F, G : \Omega \to \mathbb{R}$. We have*

$$D_k(FG) \;=\; F D_k G + G D_k F - X_k \sqrt{p_k q_k} D_k F D_k G, \quad k \geq 0,$$

and

$$D_k e^F = \frac{e^F}{-X_k \sqrt{p_k q_k}} (e^{-X_k \sqrt{p_k q_k} D_k F} - 1). \tag{30}$$

Proof. Let $F_+^k = F(S_\cdot + \mathbf{1}_{\{X_k=-1\}} \mathbf{1}_{\{k \leq \cdot\}})$ and $F_k^- = F(S_\cdot - \mathbf{1}_{\{X_k=1\}} \mathbf{1}_{\{k \leq \cdot\}})$, $k \geq 0$. We have

$$\begin{aligned}
D_k(FG) &= \frac{1}{\sqrt{p_k q_k}} (F_k^+ G_k^+ - F_k^- G_k^-) \\
&= \mathbf{1}_{\{X_k=-1\}} \frac{1}{\sqrt{p_k q_k}} \left(F(G_k^+ - G) + G(F_k^+ - F) + (F_k^+ - F)(G_k^+ - G) \right) \\
&\quad + \mathbf{1}_{\{X_k=1\}} \frac{1}{\sqrt{p_k q_k}} \left(F(G - G_k^-) + G(F - F_k^-) - (F_k - F_k^-)(G_k - G_k^-) \right) \\
&= \mathbf{1}_{\{X_k=-1\}} \left(F D_k G + G D_k F + \sqrt{p_k q_k} D_k F D_k G \right) \\
&\quad + \mathbf{1}_{\{X_k=1\}} \left(F D_k G + G D_k F - \sqrt{p_k q_k} D_k F D_k G \right).
\end{aligned}$$

We have

$$\begin{aligned}
D_k e^F &= \mathbf{1}_{\{X_k=1\}} \frac{1}{\sqrt{p_k q_k}} (e^F - e^{F_k^-}) + \mathbf{1}_{\{X_k=-1\}} \frac{1}{\sqrt{p_k q_k}} (e^{F_k^+} - e^F) \\
&= \mathbf{1}_{\{X_k=1\}} \frac{1}{\sqrt{p_k q_k}} e^F (1 - e^{-\sqrt{p_k q_k} D_k F}) + \mathbf{1}_{\{X_k=-1\}} \frac{1}{\sqrt{p_k q_k}} e^F (e^{\sqrt{p_k q_k} D_k F} - 1) \\
&= -X_k \frac{1}{\sqrt{p_k q_k}} e^F (e^{-X_k \sqrt{p_k q_k} D_k F} - 1).
\end{aligned}$$

□

The next result is the predictable representation of the functionals of $(S_n)_{n \geq 0}$. Let $\mathcal{F}_N$, $N \in \mathbb{N}$, denote the σ-field generated by $X_0, \ldots, X_N$.

Proposition 8.2 *We have the Clark formula*

$$F = E[F] + \sum_{k=1}^{\infty} E[D_k F \mid \mathcal{F}_{k-1}] Y_k, \quad F \in L^2(\Omega).$$

Proof. For $F = J_n(f_n)$ we have (see e.g. [22]):

$$\begin{aligned}
F &= J_n(f_n) = n! J_n(f_n \mathbf{1}_{\Delta_n}) = n! \sum_{k=1}^{\infty} J_{n-1}(f_n(\cdot, k) \mathbf{1}_{[1,k-1]^{n-1}}(\cdot) \mathbf{1}_{\Delta_{n-1}}(\cdot)) Y_k \\
&= n \sum_{k=1}^{\infty} J_{n-1}(f_n(\cdot, k) \mathbf{1}_{[1,k-1]^{n-1}}(\cdot)) Y_k = \sum_{k=1}^{\infty} E[D_k J_n(f_n) \mid \mathcal{F}_{k-1}] Y_k.
\end{aligned}$$

This identity also shows that $F \mapsto E[D.F \mid \mathcal{F}_{\cdot-1}]$ has norm equal to one as an operator from $L^2(\Omega)$ into $L^2(\Omega \times \mathbb{N})$:

$$\|E[D.F \mid \mathcal{F}_{\cdot-1}]\|^2_{L^2(\Omega\times\mathbb{N})} = \|F - E[F]\|^2_{L^2(\Omega)} \leq \|F - E[F]\|^2_{L^2(\Omega)} + E[F]^2 \leq \|F\|^2_{L^2(\Omega)},$$

hence the Clark formula extends to $F \in L^2(\Omega)$. $\square$

The Clark formula implies the covariance identity

$$\mathrm{Cov}(F,G) = E\left[\sum_{k=1}^{\infty} D_k F E\left[D_k G \mid \mathcal{F}_{k-1}\right]\right],$$

and we also have as in the continuous time case:

$$\mathrm{Cov}(F,G) = E\left[\sum_{k=0}^{\infty} \int_0^{\infty} e^{-s} D_k F P_s D_k G ds\right],$$

where $(P_t)_{t\in\mathbb{R}_+}$ denotes the semi-group

$$P_t F = \sum_{n=0}^{\infty} e^{-nt} J_n(f_n), \quad t \in \mathbb{R}_+,$$

$F = \sum_{n=0}^{\infty} J_n(f_n)$. The next result shows that the semi-group $(P_t)_{t\in\mathbb{R}_+}$ admits a representation by a probability kernel and an Ornstein-Uhlenbeck type process which (in the symmetric case $p_k = q_k = 1/2$, $k \in \mathbb{N}$) is in fact the Brownian motion on $\{-1,1\}^N$ considered in [2].

Proposition 8.3 *For $F \in L^2(\Omega, \mathcal{F}_N)$ we have*

$$P_t F(\omega') = \int_\Omega F(\omega) q_t^N(\omega,\omega') dP(\omega), \quad \omega,\omega' \in \Omega, \tag{31}$$

where $q_t^N(\omega,\omega')$ is the kernel

$$q_t^N(\omega,\omega') = \prod_{i=1}^{i=N} (1 + e^{-t} Y_i(\omega) Y_i(\omega')), \quad \omega,\omega' \in \Omega.$$

Proof. Since $L^2(\Omega, \mathcal{F}_N)$ is finite (2^{N+1}-)dimensional it suffices to consider the functional $Y_{k_1} \cdots Y_{k_n}$ with $(k_1, \ldots, k_n) \in \Delta_n$. We have for $\omega' \in \Omega$, $k \in \mathbb{N}$:

$$\begin{aligned} E\left[Y_k(\cdot)(1 + e^{-t} Y_k(\cdot) Y_k(\omega'))\right] &= p_k \sqrt{\frac{q_k}{p_k}} \left(1 + e^{-t}\sqrt{\frac{q_k}{p_k}} Y_k(\omega')\right) \\ &\quad -q_k \sqrt{\frac{p_k}{q_k}} \left(1 - e^{-t}\sqrt{\frac{p_k}{q_k}} Y_k(\omega')\right) \\ &= e^{-t} Y_k(\omega'), \end{aligned}$$

which implies by independence of the sequence $(X_k)_{k\in\mathbb{N}}$:

$$P_t(Y_{k_1} \cdots Y_{k_n})(\omega') = e^{-nt} Y_{k_1}(\omega') \cdots Y_{k_n}(\omega') = E[Y_{k_1} \cdots Y_{k_n} q_t^N(\cdot,\omega')], \quad \omega' \in \Omega.$$

$\square$

We have

$$\int_0^\infty e^{-t} q_t(\omega,\omega')dt = \int_0^1 q_{\log t}(\omega,\omega')dx.$$

The Ornstein-Uhlenbeck process $((X_k^t)_{k\in\mathbb{N}})_{t\in\mathbb{R}_+}$ associated to $(P_t)_{t\in\mathbb{R}_+}$ satisfies

$$P(X_k^t = 1 \mid X_k^0 = 1) = p_k + e^{-t} q_k, \qquad P(X_k^t = -1 \mid X_k^0 = 1) = q_k(1-e^{-t}),$$
$$P(X_k^t = 1 \mid X_k^0 = -1) = p_k(1-e^{-t}) \quad P(X_k^t = -1 \mid X_k^0 = -1) = q_k + e^{-t} p_k, \quad k \in \mathbb{N}.$$

In other terms, the hitting time $\tau_{1,-1} \in \mathbb{R}_+ \cup \{+\infty\}$ of -1 starting from $+1$, resp. of $+1$ starting from -1, has distribution

$$P(\tau_{1,-1} < t) = q_k(1-e^{-t}), \quad t \in \mathbb{R}_+,$$

resp.

$$P(\tau_{-1,1} < t) = p_k(1-e^{-t}), \quad t \in \mathbb{R}_+.$$

We start by showing a Gaussian deviation inequality for functionals of $(S_n)_{n\in\mathbb{N}}$.

Proposition 8.4 *Let $F : \Omega \to \mathbb{R}$ be such that $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$, and*

$$\sum_{k=0}^\infty \frac{1}{2(p_k \wedge q_k)} \|D_k F\|_\infty^2 \leq K^2.$$

Then

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2K^2}\right), \quad x \geq 0.$$

Proof. Using the inequality

$$|e^{tx} - e^{ty}| \leq \frac{1}{2} t|x-y|(e^{tx} + e^{ty}), \qquad x, y \in \mathbb{R}, \tag{32}$$

we have

$$\begin{aligned} |D_k e^{tF}| &= \frac{1}{\sqrt{p_k q_k}} |e^{tF_k^+} - e^{tF_k^-}| \leq \frac{1}{2} \frac{1}{\sqrt{p_k q_k}} t|F_k^+ - F_k^-|(e^{tF_k^+} + e^{tF_k^-}) \\ &= \frac{1}{2} t|D_k F|(e^{tF_k^+} + e^{tF_k^-}) = \frac{1}{2(p_k \wedge q_k)} t|D_k F| E\left[e^{tF} \mid X_i,\ i \neq k\right], \end{aligned}$$

and

$$\begin{aligned} E[F e^{tF}] &= E[E[D_k F \mid \mathcal{F}_k] D_k e^{tF}] \leq \sum_{k=0}^\infty \|D_k F\|_\infty E\left[|D_k e^{tF}|\right] \\ &\leq \frac{1}{2} t \sum_{k=0}^\infty \|D_k F\|_\infty^2 E\left[e^{tF_k^+} + e^{tF_k^-}\right] \\ &= t \sum_{k=0}^\infty \frac{1}{2(p_k \wedge q_k)} \|D_k F\|_\infty^2 E\left[E\left[e^{tF} \mid X_i,\ i \neq k\right]\right] \\ &= tE\left[e^{tF}\right] \sum_{k=0}^\infty \frac{1}{2(p_k \wedge q_k)} \|D_k F\|_\infty^2. \end{aligned}$$

We can conclude as in the proof of Corollary 5.4. □

In the symmetric case $p_k = q_k = 1/2$, $k \in \mathbb{N}$, we obtain

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{x^2}{2\|DF\|^2_{l^2(\mathbb{N},L^\infty(\Omega))}}\right).$$

The representation (31) implies the inequality

$$\|P_s DF\|_{L^\infty(\Omega,l^2(\mathbb{N}))} \leq \|P_s|DF|_{l^2(\mathbb{N})}\|_{L^\infty(\Omega)} \leq \|DF\|_{L^\infty(\Omega,l^2(\mathbb{N}))}, \quad s \in \mathbb{R}_+,$$

for $F \in \mathrm{Dom}(D)$, hence the following Proposition can be proved as Proposition 4.1 of Sect. 5.

Proposition 8.5 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$. Then*

$$E[e^{t(F-E[F])}] \leq \exp\left(\int_0^t h(s)ds\right), \quad 0 \leq t \leq t_0, \tag{33}$$

where h is any of the following functions:

$$h(s) = \sum_{k=0}^{\infty} \|D_k F\|_\infty \left\|e^{-sF} D_k e^{sF}\right\|_\infty, \tag{34}$$

$$h(s) = \|DF\|_{L^\infty(\Omega,l^2(\mathbb{N}))} \left\|e^{-sF} D e^{sF}\right\|_{L^\infty(\Omega,l^2(\mathbb{N}))}, \tag{35}$$

$$h(s) = \left\|\frac{e^{-sF} D e^{sF}}{DF}\right\|_\infty \|DF\|^2_{L^\infty(\Omega,l^2(\mathbb{N}))}, \qquad s \in [0, t_0]. \tag{36}$$

Although D does not satisfy the same product rule as in the continuous case, from (30) we still have the bound

$$|e^{-sF} D_k e^{sF}| \leq \frac{1}{\sqrt{p_k q_k}}(e^{s\sqrt{p_k q_k}|D_k F|} - 1), \quad k \in \mathbb{N}, \tag{37}$$

which gives the following corollary of Proposition 8.5.

Corollary 8.1 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$ for some $t_0 > 0$. Then*

$$E[e^{t(F-E[F])}] \leq \exp\left(\int_0^t h(s)ds\right), \quad 0 \leq t \leq t_0, \tag{38}$$

where h is any of the following functions:

$$h(s) = \sum_{k=0}^{\infty} \|D_k F\|_\infty \left\|\frac{1}{\sqrt{p_k q_k}}(e^{s\sqrt{p_k q_k}|D_k F|} - 1)\right\|_\infty, \tag{39}$$

$$h(s) = \|DF\|_{L^\infty(\Omega,l^2(\mathbb{N}))} \left\|\frac{1}{\sqrt{p_k q_k}}(e^{s\sqrt{p_k q_k}|D_k F|} - 1)\right\|_{L^\infty(\Omega,l^2(\mathbb{N}))}, \tag{40}$$

$$h(s) = \left\|\frac{1}{\sqrt{p_k q_k} D_k F}(e^{s\sqrt{p_k q_k}|D_k F|} - 1)\right\|_\infty \|DF\|^2_{L^\infty(\Omega,l^2(\mathbb{N}))}, \qquad s \in [0, t_0]. \tag{41}$$

Again, the inequalities given by (39), (40) and (41) are not comparable. The bound $\frac{1}{\sqrt{p_k q_k}}(e^{s\sqrt{p_k q_k}|D_k F|} - 1) \leq s|D_k F| e^{s\sqrt{p_k q_k}|D_k F|}$, $k \in \mathbb{N}$, also shows that Corollary 8.1 holds with

$$h(s) = s \sum_{k=0}^{\infty} \|D_k F\|_\infty^2 \left\| e^{s\sqrt{p_k q_k}|D_k F|} \right\|_\infty,$$

and

$$h(s) = s\|DF\|_{L^\infty(\Omega, l^2(\mathbb{N}))} \|e^{s\sqrt{p_\cdot q_\cdot}|D_\cdot F|} D_\cdot F\|_{L^\infty(\Omega, l^2(\mathbb{N}))}, \qquad s \in [0, t_0].$$

The following corollary is obtained with the same proof as on the Poisson space.

Corollary 8.2 *Let $F \in \mathrm{Dom}(D)$ be such that $E[e^{t_0|F|}] < \infty$, $\sqrt{p_k q_k}|D_k F| \leq K$, $k \in \mathbb{N}$, for some $K \geq 0$, and $\|DF\|_{L^\infty(\Omega, l^2(\mathbb{N}))} < \infty$. Then*

$$\begin{aligned} P(F - E[F] \geq x) \quad &\leq \quad \exp\left(-\frac{\|DF\|^2_{L^\infty(\Omega, l^2(\mathbb{N}))}}{K^2} g\left(\frac{xK}{\|DF\|^2_{L^\infty(\Omega, l^2(\mathbb{N}))}} \right) \right) \\ &\leq \quad \exp\left(-\frac{x}{2K} \log\left(1 + \frac{xK}{\|DF\|^2_{L^\infty(\Omega, l^2(\mathbb{N}))}} \right) \right), \end{aligned}$$

with $g(u) = (1+u)\log(1+u) - u$, $u \geq 0$.

Proof. We use the inequality

$$-s \leq \frac{e^{-sF} D_k e^{sF}}{D_k F} = \frac{1}{-X_k \sqrt{p_k q_k} D_k F}(e^{-sX_k\sqrt{p_k q_k} D_k F} - 1) \leq \frac{e^{sK} - 1}{K},$$

and apply Corollary 8.1. □

In case $p_k = p$ and $q_k = q$ for all $k \in \mathbb{N}$, the conditions $\sqrt{pq}|D_k F| \leq \beta$, $k \in \mathbb{N}$, and $\|DF\|^2_{L^\infty(\Omega, l^2(\mathbb{N}))} \leq \alpha^2$, give

$$P(F - E[F] \geq x) \leq \exp\left(-\frac{\alpha^2 pq}{\beta^2} g\left(\frac{x\beta}{\alpha^2 pq} \right) \right) \leq \exp\left(-\frac{x}{2\beta} \log\left(1 + \frac{x\beta}{\alpha^2 pq} \right) \right),$$

which is Relation (13) in [4]. In particular if F is $\mathcal{F}_N$-measurable, then

$$P(F - E[F] \geq x) \leq \exp\left(-Ng\left(\frac{x}{\beta N} \right) \right) \leq \exp\left(-\frac{x}{\beta}\left(\log\left(1 + \frac{x}{\beta N} \right) - 1 \right) \right).$$

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