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Besov regularity for the indefinite Skorohod integral with respect to the fractional Brownian motion: the singular case

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Abstract

Using the techniques of the Malliavin calculus and the properties of Gaussian processes, we prove that the paths of the indefinite Skorohod integral with respect to the fractional Brownian motion with Hurst parameter less than $\frac{1}{2}$ belongs to the Besov space $\mathcal{B}^{H}_{p,\infty}$, for any $p > \frac{1}{H}$.

Key words: Fractional Brownian motion, Stochastic integrals, Malliavin calculus.
Mathematics Subject Classification : 60H05, 60H07.

1 Introduction

Let $(B^H_t)_{t\in[0,1]}$ a fractional Brownian motion (fBm) with Hurst parameter $H \in (0,1)$. This process is Gaussian, it starts from zero and its covariance it given by

$$R(t,s) = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right)$$

The main problem raised by the fBm and related processes is that they are not Markovian, even more they have not any semimartingale property, hence it is somewhat difficult to develop a stochastic integration with respect to fBm.

Recently, there has been a development in the stochastic calculus with respect to this process. We will focus our attention to the Skorohod integral introduced in [2], which is in fact a transfer of the Skorohod integration with respect to the Brownian motion to the Reproducing Kernel Hilbert Space of the fractional Brownian motion.

Let $(u_t)_{t\in[0,1]}$ a stochastic process integrable with respect to B^H and consider the indefinite integral $\int_0^t u_s dB_s^H$. It was proved in [2] that its paths are Hölder continuous of order $\delta < H$. Moreover, it is well-known (see [5]) that B^H itself belongs to the Besov spaces $\mathcal{B}^H_{p,\infty}$, for every $p > \frac{1}{H}$. A natural question is to see if the Skorohod integral $\int_0^t u_s dW_s$ keeps the same Besov regularity as the fractional Brownian motion. This problem was studied in [9] in the regular case $H > \frac{1}{2}$, using an intrinsec Malliavin calculus with respect to fBm. For the study of the Besov regularity of a Skorohod integral on the Wiener space we refer to [4] and [6].

The aim of this work is to prove that also in the singular case ($H < \frac{1}{2}$) the trajectories of the Skorohod integral process with respect to B^H belong to the Besov space $\mathcal{B}^H_{p,\infty}$, for every $p > \frac{1}{H}$. Due to the singularity of the kernel of the fractional Brownian motion for $H < \frac{1}{2}$, we will need to assume some uniformly regularity conditions on the integrand u and on its Malliavin derivative.

The organization of the paper is as follows: Section 2 contains some preliminaries on the Malliavin calculus, fractional Brownian motion and Besov spaces, and in Section 3 we study the regularity of the indefinite stochastic integral.

2 Preliminaries

2.1 Malliavin calculus

Let $T = [0, 1]$ the unit interval and consider $(W_t)_{t\in T}$ a standard Wiener process on the probability space $(\Omega, \mathcal{F}, P)$. By $\mathcal{S}$ we will denote the set of smooth random variables on $(\Omega, \mathcal{F}, P)$, that is, every $F \in \mathcal{S}$ has the form

$$F = f\left(W_{t_1}, \ldots, W_{t_n}\right) \tag{1}$$

with $t_1, \ldots, t_n \in T$ and $f \in C_p^\infty$ (f is infinitely continuously differentiable function on R^n such that f and all of its derivatives has polynomial growth). We introduce the Malliavin derivative of $F \in \mathcal{S}$ of the form (1) as

$$D_t F = \sum_{i=1}^n \frac{\partial f}{\partial x_i}\left(W_{t_1}, \ldots, W_{t_n}\right) 1_{[0,t_i]}(t),\ t \in T$$

The operator D from $L^2(\Omega)$ to $L^2(T \times \Omega)$ is closable and its domain (denoted $D^{1,2}$) is the closure of $\mathcal{S}$ with respect to the norm

$$\|F\|_{1,2}^2 = E\left|F\right|^2 + E\|DF\|^2_{L^2(T)}$$

More generally, we can introduce, for k integer and p real, the space $D^{k,p}$ of Malliavin differentiable random variables as the closure of $\mathcal{S}$ with respect to the seminorm

$$\|F\|_{k,p}^p = E\left|F\right|^p + \sum_{j=1}^k E\|D^{(j)}F\|^p_{L^2(T)}$$

where the multiple derivation $D^{(j)}$ is defined iteratively. The adjoint of D, denoted by δ is defined on the domain

$$Dom(\delta) = \left\{ u \in L^2(T \times \Omega) / \left| E \int_T u_s D_s F ds \right| \leq C \|F\|_{L^2(\Omega)} \right\}$$

and it is given by the duality relationship

$$E(F\delta(u)) = E \int_T u_s D_s F ds \ , u \in Dom(\delta), F \in \mathcal{S}$$

We also use the notation $\delta(u) = \int_T u_s dW_s$ and we call $\delta(u)$ the Skorohod integral of u. By $L^{k,p}$ we will denote the Banach space $L^p\left(T^k; D^{k,p}\right)$ and this space contains Skorohod integrable processes $(L^{1,p} \subset Dom(\delta))$. We will use the Meyer inequalities

$$\|\delta(u)\|_p \leq \|u\|_{1,p} \tag{2}$$

and the integration by parts formula for the Skorohod integral

$$F\delta(u) = \delta(Fu) + \int_0^1 D_s F u_s ds \tag{3}$$

for every $F \in D^{1,2}$, $u \in Dom(\delta)$ such that $E\left(F^2 \int_T u_s^2 ds\right) < \infty$. Recall also the commutativity relationship between the derivative operator and the Skorohod integral

$$D_t \delta(u) = u_t + \delta(D_t u) \tag{4}$$

for every $t \in T$, $u \in Dom(\delta)$ with $D_t u \in Dom(\delta)$. Note that the properties (3) and (4) hold for $u \in L^{2,2}$.

2.2 Fractional Brownian motion

Fix $T = [0,1]$ the unit interval and consider $(B_t^H)_{t \in T}$ the fractional Brownian motion with Hurst parameter $H \in (0,1)$. That means, B^H is a centered Gaussian process with covariance

$$R(t,s) = E(B_s^H B_t^H) = \frac{1}{2}(t^{2H} + s^{2H} - |t-s|^{2H})$$

The process B^H admits an integral representation of the form

$$B_t^H = \int_0^t K(t,s) dW_s,$$

where $W = \{W_t, t \in [0,T]\}$ is a Wiener process, and $K(t,s)$ is the kernel

$$K(t,s) = c_H \left(t-s\right)^{H-\frac{1}{2}} + s^{H-\frac{1}{2}} F_1\left(\frac{t}{s}\right), \tag{5}$$

for every $t \geq s$, c_H being a constant and

$$F_1(z) = c_H \left(\frac{1}{2} - H\right) \int_0^{z-1} \theta^{H-\frac{3}{2}} \left(1 - (\theta+1)^{H-\frac{1}{2}}\right) d\theta.$$

We also put by convention $K(t,s) = 0$ if $s > t$. This kernel satisfies the condition :

$$\frac{\partial K}{\partial t}(t,s) = c_H (H - \frac{1}{2}) (\frac{s}{t})^{\frac{1}{2}-H} (t-s)^{H-\frac{3}{2}}. \tag{6}$$

We will denote by $\mathcal{H}$ the Reproducing kernel Hilbert space of the fBm . In fact $\mathcal{H}$ is the closure of set of indicator functions $\left\{1_{[0,t]}, t \in T\right\}$ with respect to the scalar product

$$\langle 1_{[0,t]}, 1_{[0,s]} \rangle_{\mathcal{H}} = R(t,s)$$

The mapping $1_{[0,t]} \to B_t$ provides an isometry between $\mathcal{H}$ and the first Wiener chaos and we will denote by $B(\phi)$ the image of $\phi \in \mathcal{H}$ by the previous isometry. In this way, the $\mathcal{H}-$ indexed process $(B(\phi))_{\phi \in \mathcal{H}}$ becomes a centered Gaussian process such that $E(B(\phi)B(h)) = \langle \phi, h \rangle_{\mathcal{H}}$. A such process is called an isonormal process and we can define a Skorohod integral with respect to such processes (see [8]). We will recall only that the relation between the Skorohod integral with respect to B^H and the Skorohod integral with respect to W is given by

$$\int_0^t u_s dB_s^H = \int_0^t u_s K(t,s) dW_s + \int_0^t \left(\int_s^t (u_r - u_s) \frac{\partial K}{\partial r}(r,s) dr\right) dW_s. \tag{7}$$

In the case $H > \frac{1}{2}$, this integral can be also written

$$\int_0^t u_s dB_s^H = \int_0^t \left(\int_s^t u_r \frac{\partial K}{\partial r}(r,s) dr\right) dW_s.$$

We refer to [2] for a complete exposition of the stochastic calculus with respect to B^H.

2.3 Besov spaces

Denote by $L^p(T)$ the space of Lebesgue integrable functions with exponent p $(1 \leq p < \infty)$. Let $f : T \to R$ be a function in $L^p(T)$, one can measure its smoothness by its modulus of continuity computed in L^p-norm. For this end let us define for any $t \in I$,

$$\omega_p(f,t) = \sup_{|h| \leq t} \left(\int_{T_h} |f(x+h) - f(x)|^p \, dx\right)^{\frac{1}{p}},$$

where $T_h = \{x \in T,\ x+h \in I\}$, and we will denote

$$\|f\|_{\alpha,p,\infty} := \|f\|_{L^p(T)} + \sup_{0<t\leq 1} \frac{\omega_p(f,t)}{t^\alpha}.$$

The Besov space $B^\alpha_{p,\infty}$ is the class of functions f in L^p such that $\|f\|_{\alpha,p,\infty} < \infty$; $B^\alpha_{p,\infty}$ endowed with the norm $\|.\|_{\alpha,p,\infty}$ is a non-separable Banach space. We will consider a separable subspace $B^{\alpha,0}_{p,\infty}$ of $B^\alpha_{p,\infty}$ formed with the functions $f \in B^\alpha_{p,\infty}$ satisfying $\omega_p(f,t) = \mathrm{o}(t^\alpha)$ (as $t \downarrow 0$). To prove our main results we need the characterization of the Besov spaces in terms of the coefficients of the expansion of a continuous functions in the basis consisting of Schauder functions. Let f be a continuous function, let us note by $\{C_n(f),\ n \geq 0\}$ the coefficients of the decomposition of f in the Schauder basis giving by (see [5]):

$$\begin{cases} C_0(f) = f(0),\ \ C_1(f) = f(1) - f(0), \\ \\ \text{and for } n = 2^j + k,\ j \in N \text{ and } k = 0, ..., 2^j - 1, \\ \\ C_n(f) = 2.2^{\frac{j}{2}}\{f(\frac{2k-1}{2^{j+1}}) - \frac{1}{2}\left[f(\frac{2k}{2^{j+1}}) + f(\frac{2k-2}{2^{j+1}})\right]\}. \end{cases}$$

Let $0 < \alpha < 1$. We will use the following characterization theorem.

Theorem 1 *Let $p \geq 1$ and $0 < \alpha < 1$.*
1. If $\alpha > \frac{1}{p}$, then $B^\alpha_{p,\infty}$ is the space of continuous functions, linearly isomorphic to some sequences space, and we have the following equivalence of norms:

$$\|f\|_{\alpha,p,\infty} \sim \max\left\{|C_0(f)|,\ |C_1(f)|,\ \sup_{j\geq 0} 2^{-j(\frac{1}{2}-\alpha+\frac{1}{p})}\left[\sum_{n=2^j+1}^{2^{j+1}} |C_n(f)|^p\right]^{\frac{1}{p}}\right\},$$

2. f belongs to $B^{\alpha,0}_{p,\infty}$ if and only if

$$\lim_{j\to\infty} 2^{-j(\frac{1}{2}-\alpha+\frac{1}{p})}\left[\sum_{n=2^j+1}^{2^{j+1}} |C_n(f)|^p\right]^{\frac{1}{p}} = 0.$$

For the proof of this result we refer to [5].

3 Besov regularity of the indefinite Skorohod integral with respect to the fractional Brownian motion

Fix $H < \frac{1}{2}$ and let $u = (u_s)_{s\in[0,1]}$ a stochastic process Skorohod integrable with respect to the fBm . Put

$$X_t = \int_0^t u_s dB^H_s$$

$$= \int_0^t u_s K(t,s) dW_s + \int_0^t \left(\int_s^t (u_r - u_s)\frac{\partial K}{\partial r}(r,s)dr\right) dW_s$$

$$= Y_t + Z_t$$

We will discuss first the Besov regularity of the first term Y_t because this one gives the main result. The regularity of the second summand Z_t can be easily studied by assuming Hölder continuity conditions on the integrand u (see Theorem 2). We define

$$K_{j,k}(\cdot, s) = 2^{\frac{j}{2}} \left[2K(\frac{2k-1}{2^{j+1}}, s) - K(\frac{k-1}{2^j}, s) - K(\frac{k}{2^j}, s) \right]$$

and

$$B_{j,k} = \int_0^1 K_{j,k}(\cdot, s) dW_s.$$

Since $K(t,s) = 0$ for $t < s$, we can write $B_t^H = \int_0^1 K(t,s)dW_s$ and thus $B_{j,k}$ are the coefficients of the decomposition of the fBm in the Schauder basis. We know (see [5]) that $(B_{j,k})_{j,k}$ is a Gaussian sequence with

$$E(B_{j,k}^2) = \int_0^1 K_{j,k}^2(\cdot, s) ds = (2^{2-2H} - 1) 2^{j(1-2H)} \tag{8}$$

and

$$|E(B_{j,k} B_{j,k'})| \le C \frac{2^{j(1-2H)}}{1 + |k' - k|^{4-2H}} \tag{9}$$

Since $Y_t = \int_0^1 K(t,s) u_s dW_s$, its coefficients will be

$$Y_{j,k}(1) = \int_0^1 K_{j,k}(\cdot, s) u_s dW_s.$$

To prove that $t \to Y_t$ belongs to $\mathcal{B}^H_{p,\infty}$, by Theorem 1 it suffices that, almost surely

$$\sup_{j \ge 1} 2^{-2jp\left(\frac{1}{2} - H + \frac{1}{2p}\right)} \sum_k |Y_{j,k}(1)|^{2p} < \infty \tag{10}$$

for any $p > \frac{1}{H}$. We begin with the following lemma .

Lemma 1 *Let $u \in L^{1,2}$ satisfying*

$$\sup_{s \in T} |u_s| < C \text{ and } \sup_{s \in T} \left(\int_0^1 (D_r u_s)^2 dr \right) \le C \tag{11}$$

where C is a positive constant. Then, for any p integer, we have

$$E|Y_{j,k}(1)|^{2p} \le c(p,H) E|B_{j,k}|^{2p} \le c(p,H) 2^{jp(1-2H)}$$

Proof: We show that

$$E|Y_{j,k}(1)|^{2p} \le c(p) \left(\int_0^1 K_{j,k}^2(\cdot, s) ds \right)^p$$

and the conclusion follows from (8). We can estimate the L^{2p}-norm of $Y_{j,k}$ by Meyer's inequality (2)

$$\begin{aligned}
E|Y_{j,k}(1)|^{2p} &\leq E\left(\int_0^1 u_s^2 K_{j,k}^2(\cdot,s)ds\right)^p \\
&+ E\left(\int_0^1\int_0^1 K_{j,k}^2(\cdot,s)(D_r u_s)^2 dr ds\right)^p \\
&\leq E\left(\sup_s |u_s|\right)^{2p}\left(\int_0^1 K_{j,k}^2(\cdot,s)ds\right)^p \\
&+ E\left(\sup_s \int_0^1 (D_r u_s)^2 dr\right)^p \left(\int_0^1 K_{j,k}^2(\cdot,s)ds\right)^p
\end{aligned}$$

So, under conditions (11) and by (8), we obtain the conclusion. □

We use now the property (3) and (4) of the Skorohod integral and we obtain

$$\begin{aligned}
(Y_{j,k}(1))^{2p} &= \int_0^1 Y_{j,k}^{2p-1}(1)K_{j,k}(\cdot,s)u_s dW_s + \int_0^1 D_s\left[Y_{j,k}^{2p-1}(1)\right]K_{j,k}(\cdot,s)u_s ds \\
&= \int_0^1 Y_{j,k}^{2p-1}(1)K_{j,k}(\cdot,s)u_s dW_s + (2p-1)Y_{j,k}^{2p-2}(1)\int_0^1 K_{j,k}^2(\cdot,s)u_s^2 ds \\
&+ (2p-1)Y_{j,k}^{2p-2}(1)\int_0^1 K_{j,k}(\cdot,s)u_s\delta\left(D_s u_\star K_{j,k}(\cdot,\star)\right)ds \\
&= A_{j,k}, + B_{j,k} + C_{j,k}
\end{aligned}$$

("$\star$" denotes the variable of the Skorohod integral). Thus, the left side of (10) can be written as

$$\begin{aligned}
&\sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}\sum_k |Y_{j,k}(1)|^{2p} \\
&= \sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}\sum_k (A_{j,k}+B_{j,k}+C_{j,k}) \\
&= A+B+C
\end{aligned}$$

We will estimate the three terms appearing in the above expression.

3.1 Estimation of the term A

We need to prove that, P-a. s.

$$\sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}\sum_k \int_0^1 Y_{j,k}^{2p-1}(1)K_{j,k}(\cdot,s)u_s dW_s < \infty$$

By Borel -Cantelli Lemma and Tchebyschev inequalities, it suffices to prove that

$$\sum_{j\geq 1} 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} E\left[\int_0^1 \sum_k Y_{j,k}^{2p-1}(s) K_{j,k}(\cdot,s) u_s dW_s\right]^2 < \infty \qquad (12)$$

Since

$$E\left[\int_0^1 \sum_k Y_{j,k}^{2p-1}(1) K_{j,k}(\cdot,s) u_s dW_s\right]^2$$
$$\leq E\int_0^1 \left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1) K_{j,k}(\cdot,s) u_s\right]^2 ds + E\int_0^1\int_0^1 \left(D_\alpha\left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1) K_{j,k}(\cdot,s) u_s\right]\right)^2 d\alpha ds$$

we can bound the expression (12) by

$$\sum_{j\geq 1} 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} E\int_0^1 \left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1) K_{j,k}(\cdot,s) u_s\right]^2 ds$$
$$+\sum_{j\geq 1} 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} E\int_0^1\int_0^1 \left(D_\alpha\left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1) K_{j,k}(\cdot,s) u_s\right]\right)^2 d\alpha ds$$
$$= \sum_{j\geq 1} 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} (A_j^{(1)} + A_j^{(2)}). \qquad (13)$$

The following result give the convergence of the first sum in (13).

Proposition 1 *Let $u \in L^\infty([0,1]\times\Omega)\cap L^{1,2}$. For any integer p it holds that*

$$\sum_{j\geq 1} 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} A_j^{(1)} < \infty \qquad (14)$$

where

$$A_j^{(1)} = E\int_0^1 \left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1) K_{j,k}(\cdot,s) u_s\right]^2 ds$$

Proof: Since $u \in L^\infty([0,1]\times\Omega)$, we will have

$$E\int_0^1 \left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1)K_{j,k}(\cdot,s)u_s\right]^2 ds$$

$$\leq \|u\|_\infty^2 E\int_0^1 \sum_{k,k'} Y_{j,k}^{2p-1}(1)Y_{j,k'}^{2p-1}(1)K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds$$

$$= \|u\|_\infty^2 E\int_0^1 \sum_{k} Y_{j,k}^{4p-2}(1)K_{j,k}^2(\cdot,s)ds$$

$$+\|u\|_\infty^2 E\int_0^1 \sum_{k\neq k'} Y_{j,k}^{2p-1}(1)Y_{j,k'}^{2p-1}(1)K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds$$

$$\leq \|u\|_\infty^2 \sum_k E\left|Y_{j,k}^{4p-2}(1)\right|\left(\int_0^1 K_{j,k}^2(\cdot,s)ds\right)$$

$$+\|u\|_\infty^2 \sum_{k\neq k'} E\left|Y_{j,k}^{2p-1}(1)Y_{j,k'}^{2p-1}(1)\right|\left|\int_0^1 K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right|$$

By Lemma 1 and Hölder's inequalities, the last expression can be majorized by

$$c(p,H)2^{j(2p-1)(1-2H)}\sum_k\left(\int_0^1 K_{j,k}^2(\cdot,s)ds\right)$$

$$+c(p,H)2^{j(2p-1)(1-2H)}\sum_{k\neq k'}\left|\int_0^1 K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right|$$

Since , by (8) and (9)

$$\int_0^1 K_{j,k}^2(\cdot,s)ds = E(B_{j,k}^2) = c(H)2^{j(1-2H)}$$

and

$$\left|\int_0^1 K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right| = |E(B_{j,k}B_{j,k'})| \leq C\frac{2^{j(1-2H)}}{1+|k'-k|^{4-2H}}$$

it remains to observe that

$$\sum_{k\neq k'}\frac{1}{1+|k'-k|^{4-2H}} \leq 2\sum_{k'>k}\frac{1}{1+|k'-k|^{4-2H}} \leq 2\sum_{k=1}^{2^j}\sum_{l=1}^{2^j-k}\frac{1}{1+l^{4-2H}} \leq C2^j$$

because $\sum_{l=1}^\infty \frac{1}{1+l^{4-2H}}$ is convergent. Finally, expression (14) is less than

$$c(p,H)\sum_{j\geq1}2^{-j} < \infty. \square$$

Now in order to finish the finitude of the the term A , we need to estimate the second summand of (13).

Proposition 2 *Let $u \in L^\infty([0,1]\times\Omega)\cap L^{2,2}$. Moreover, suppose that*

$$\sup_{\omega,r,\alpha}(D_\alpha u_r)^2 < M \text{ and } \sup_{\omega,r,\alpha}\int_0^1 (D_\beta D_\alpha u_r)^2 d\beta < M \tag{15}$$

for some constant $M>0$. For any integer p it holds that

$$\sum_{j\geq 1} 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} A_j^{(2)} \tag{16}$$

with

$$A_j^{(2)} = E\int_0^1\int_0^1 \left(D_\alpha\left[\sum_{k=1}^{2^j} Y_{j,k}^{2p-1}(1)K_{j,k}(\cdot,s)u_s\right]\right)^2 d\alpha ds < \infty$$

Proof: Using the commutation between the Skorohod integral and the Malliavin derivative (4) we obtain

$$\begin{aligned} D_\alpha\left[Y_{j,k}^{2p-1}(1)u_s\right] &= (D_\alpha u_s)Y_{j,k}^{2p-1}(1) + (2p-1)u_s Y_{j,k}^{2p-2}(1)D_\alpha Y_{j,k} \\ &= (D_\alpha u_s)Y_{j,k}^{2p-1}(1) + (2p-1)u_s Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,\alpha)u_\alpha \\ &+ (2p-1)u_s Y_{j,k}^{2p-2}(1)\delta\left(D_\alpha u_\star K_{j,k}(\cdot,\star)\right) \end{aligned}$$

and, therefore we will bound $A_j^{(2)}$ as follows

$$\begin{aligned} A_j^{(2)} &\leq c(p)E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)D_\alpha u_s Y_{j,k}^{2p-1}(1)\right)^2 d\alpha ds \\ &+ c(p)E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)u_s Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,\alpha)u_\alpha\right)^2 d\alpha ds \\ &+ c(p)E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)u_s u_\alpha Y_{j,k}^{2p-2}(1)\delta\left(D_\alpha u_\star K_{j,k}(\cdot,\star)\right)\right)^2 d\alpha ds \\ &= c(p)\left(a_j^{(1)}+a_j^{(2)}+a_j^{(3)}\right) \end{aligned}$$

Next, we regard the term $a_j^{(1)}$. It holds that

$$a_j^{(1)} = E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)D_\alpha u_s Y_{j,k}^{2p-1}(1)\right)^2 d\alpha ds$$

$$\leq \sup_{\omega,\alpha,s}(D_\alpha u_s)^2 E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)Y_{j,k}^{2p-1}(1)\right)^2 d\alpha ds$$

$$\leq ME\int_0^1\int_0^1\sum_{k=1}^{2^j}K_{j,k}^2(\cdot,s)Y_{j,k}^{4p-2}(1)d\alpha ds$$

$$+M\sum_{k\neq k'}\left(\int_0^1\int_0^1 K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)d\alpha ds\right)E\left(Y_{j,k}^{2p-1}(1)Y_{j,k'}^{2p-1}(1)\right)$$

$$\leq\sum_k E\left|Y_{j,k}^{4p-2}(1)\right|\left(\int_0^1 K_{j,k}^2(\cdot,s)ds\right)$$

$$+\sum_{k\neq k'}E\left|Y_{j,k}^{2p-1}(1)Y_{j,k'}^{2p-1}(1)\right|\left|\int_0^1 K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right|$$

Then by the same arguments as before, we will obtain that

$$\sum_{j\geq 1}2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}a_j^{(1)}\leq c(p,H)\sum_j 2^{-j}<\infty$$

Concerning the summand $a_j^{(2)}$, we can write

$$E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)u_sY_{j,k}^{2p-2}(1)K_{j,k}(\cdot,\alpha)u_\alpha\right)^2 d\alpha ds$$

$$=E\int_0^1\int_0^1 u_\alpha^2u_s^2\sum_{k=1}^{2^j}K_{j,k}^2(\cdot,s)Y_{j,k}^{4p-4}(1)K_{j,k}^2(\cdot,\alpha)d\alpha ds$$

$$+E\int_0^1\int_0^1 u_\alpha^2u_s^2\sum_{k\neq k'}K_{j,k}(\cdot,s)Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,\alpha)K_{j,k'}(\cdot,s)Y_{j,k'}^{2p-2}(1)K_{j,k'}(\cdot,\alpha)d\alpha ds$$

$$\leq\|u\|_\infty^4\sum_k\left(\int_0^1 K_{j,k}^2(\cdot,s)ds\right)^2E\left|Y_{j,k}^{4p-4}(1)\right|$$

$$+\|u\|_\infty^4\sum_{k\neq k'}\left(\int_0^1 K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right)^2E\left|Y_{j,k}^{2p-2}(1)Y_{j,k'}^{2p-2}(1)\right|$$

Once again, Lemma 1 will imply that

$$\sum_{j\geq 1}2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}a_j^{(2)}<\infty$$

It remains to estimate the term $a_j^{(3)}$. Denote by

$$\delta\left(D_\alpha u_\star K_{j,k}(\cdot,\star)\right)=v_{j,k}(\alpha)$$

We use the same type of calculus as before

$$\begin{aligned}
a_j^{(3)} &= E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)u_sY_{j,k}^{2p-2}(1)\delta\left(D_\alpha u_\star K_{j,k}(\cdot,\star)\right)\right)^2 d\alpha ds\\
&\le \|u\|_\infty^2 E\int_0^1\int_0^1\left(\sum_{k=1}^{2^j}K_{j,k}(\cdot,s)Y_{j,k}^{2p-2}(1)\delta\left(D_\alpha u_\star K_{j,k}(\cdot,\star)\right)\right)^2 d\alpha ds\\
&= \|u\|_\infty^2 E\int_0^1\int_0^1\sum_{k=1}^{2^j}K_{j,k}^2(\cdot,s)Y_{j,k}^{4p-4}(1)\left(\delta\left(D_\alpha u_\star K_{j,k}(\cdot,\star)\right)\right)^2 d\alpha ds\\
&+ \|u\|_\infty^2 E\int_0^1\int_0^1\sum_{k\neq k'}K_{j,k}(\cdot,s)Y_{j,k}^{2p-2}(1)v_{j,k}(\alpha)K_{j,k'}(\cdot,s)Y_{j,k'}^{2p-2}(1)v_{j,k'}(\alpha)dsd\alpha\\
&\le \|u\|_\infty^2\sum_{k=1}^{2^j}E\int_0^1\int_0^1 K_{j,k}^2(\cdot,s)Y_{j,k}^{4p-4}(1)v_{j,k}^2(\alpha)d\alpha ds\\
&+ \|u\|_\infty^2\sum_{k\neq k'}\left|\int_0^1K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right| E\left|Y_{j,k'}^{2p-2}(1)Y_{j,k'}^{2p-2}(1)\int_0^1\left(v_{j,k}(\alpha)v_{j,k'}(\alpha)\right)d\alpha\right|\\
&\le \|u\|_\infty^2\sum_{k=1}^{2^j}\left(\int_0^1K_{j,k}^2(\cdot,s)ds\right)E\left(Y_{j,k}^{4p-4}(1)\int_0^1v_{j,k}^2(\alpha)d\alpha\right)\\
&+ \|u\|_\infty^2\sum_{k\neq k'}\left|\int_0^1K_{j,k}(\cdot,s)K_{j,k'}(\cdot,s)ds\right|\left(E\left|Y_{j,k'}^{4p-4}(1)Y_{j,k'}^{4p-4}(1)\right|\right)^{\frac12}\\
&\times \int_0^1\left(E\left|v_{j,k}^2(\alpha)v_{j,k'}^2(\alpha)\right|\right)^{\frac12}d\alpha\\
&\le \|u\|_\infty^2\sum_{k=1}^{2^j}\left(\int_0^1K_{j,k}^2(\cdot,s)ds\right)\left(E\left|Y_{j,k}^{8p-8}(1)\right|\right)^{\frac12}\int_0^1\left(E\left|v_{j,k}^4(\alpha)\right|\right)^{\frac14}\left(E\left|v_{j,k'}^4(\alpha)\right|\right)^{\frac14}d\alpha
\end{aligned}$$

Moreover, from Meyer's inequalities

$$\begin{aligned}
E(v_{j,k}(\alpha))^4 &\le E\left(\int_0^1(D_\alpha u_r)^2K_{j,k}^2(\cdot,r)dr\right)^2\\
&+ E\left(\int_0^1\int_0^1(D_\beta D_\alpha u_r)^2K_{j,k}^2(\cdot,r)drd\beta\right)^2\\
&\le \int_0^1K_{j,k}^2(\cdot,r)\left[\sup_{\omega,r,\alpha}(D_\alpha u_r)^2+\sup_{\omega,r,\alpha}\int_0^1(D_\beta D_\alpha u_r)^2d\beta\right]dr\\
&\le c(p,H)2^{2j(1-2H)}
\end{aligned}$$

Therefore, by our conditions (15) we obtain that

$$\sum_{j\ge1}2^{-4jp\left(\frac12-H+\frac{1}{2p}\right)}a_j^{(3)}<\infty\ \Box.$$

3.2 The estimation of the term B

In this section we will show that, almost surely

$$\sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} \sum_k \int_0^1 Y_{j,k}^{2p-2}(1) K_{j,k}(\cdot,s) u_s v_{j,k}(s) ds < \infty \tag{17}$$

with $\delta\left(D_s u_\star K_{j,k}(\cdot,\star)\right) = v_{j,k}(s)$. To this end, using Tchebyschev inequalities and Borel-Cantelli Lemma, we will prove that

$$\sum_j 2^{-4jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} E\left(\sum_k \int_0^1 Y_{j,k}^{2p-2}(1) K_{j,k}(\cdot,s) u_s v_{j,k}(s) ds\right)^2 < \infty \tag{18}$$

Assume in this section that u and Du are uniformly bounded : $u \in L^\infty(T\times\Omega)$ and

$$\sup_{\omega,\alpha,s} (D_\alpha u_s)^2 \leq K$$

Since

$$v_{j,k}(s) = \int_0^1 D_s u_\alpha K_{j,k}(\cdot,\alpha) dW_\alpha$$

we can write, by (3) and Fubini anticipating theorem,

$$\begin{aligned}
\int_0^1 K_{j,k}(\cdot,s) u_s v_{j,k}(s) ds &= \int_0^1 \left(\int_0^1 K_{j,k}(\cdot,s) D_s u_\alpha ds\right) K_{j,k}(\cdot,\alpha) dW_\alpha \\
&= \int_0^1 w_{j,k}(\alpha) K_{j,k}(\cdot,\alpha) dW_\alpha \\
&+ \int_0^1 \int_0^1 K_{j,k}(\cdot,s) K_{j,k}(\cdot,\alpha) D_s u_\alpha D_\alpha u_s ds d\alpha
\end{aligned}$$

with the notation

$$w_{j,k}(\alpha) = \int_0^1 K_{j,k}(\cdot,s) u_s D_s u_\alpha ds$$

The mean appearing in the left side of (18) is bounded by

$$2E\int_0^1 \left(\sum_k Y_{j,k}^{2p-2}(1) K_{j,k}(\cdot,s) w_{j,k}(s)\right)^2 ds$$

$$+2E\int_0^1\int_0^1 \left(D_r\left[\sum_k Y_{j,k}^{2p-2}(1) K_{j,k}(\cdot,s) w_{j,k}(s)\right]\right)^2 dr ds$$

$$+2E\left(\sum_k Y_{j,k}^{2p-2}(1) \int_0^1\int_0^1 K_{j,k}(\cdot,s) K_{j,k}(\cdot,\alpha) D_s u_\alpha D_\alpha u_s ds d\alpha\right)^2$$

In fact, the estimation of the first two terms from above can be done in the same way as for the term A. We will compute only the first term. Also, concerning the last summand, we refer to the proof of the convergence of $a_j^{(2)}$. It holds

$$\begin{aligned}
&E\int_0^1 \left(\sum_k Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,s)w_{j,k}(s)\right)^2 ds\\
&= E\int_0^1 \left[\int_0^1 D_s u_\alpha \sum_k Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,s)K_{j,k}(\cdot,\alpha)d\alpha\right]^2 ds\\
&\leq \sup_{\alpha,s}(D_\alpha u_s)^2 E\int_0^1\int_0^1 \left(\sum_k Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,s)K_{j,k}(\cdot,\alpha)\right)^2 d\alpha ds\\
&\leq KE\int_0^1\int_0^1 \sum_k Y_{j,k}^{4p-4}(1)K_{j,k}^2(\cdot,s)K_{j,k}^2(\cdot,\alpha)d\alpha ds\\
&+KE\int_0^1\int_0^1 \sum_{k\neq k'} Y_{j,k}^{2p-2}(1)Y_{j,k'}^{2p-2}(1)K_{j,k}(\cdot,s)K_{j,k}(\cdot,\alpha)K_{j,k'}(\cdot,s)K_{j,k'}(\cdot,\alpha)d\alpha ds\\
&\leq c(p,H)2^{2j(1-2H)}2^j 2^{j(2p-2)(1-2H)}\\
&+c(p,H)\sum_{k\neq k'}\left|E(B_{j,k}B_{j,k'})\right|^2 2^{j(2p-2)(1-2H)}
\end{aligned}$$

and observe that from (9), the relation (18) is proved. By the previous computations, we can state the result

Proposition 3 *If u is a stochastic processes belonging to $L^\infty([0,1]\times\Omega)\cap L^{2,2}$ such that there exists a constant $M>0$ with $\sup_{\omega,\alpha,s}(D_\alpha u_s)^2\leq M$, we have that*

$$\sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}\sum_k\int_0^1 Y_{j,k}^{2p-2}(1)K_{j,k}(\cdot,s)u_s v_{j,k}(s)ds<\infty \ a.s.$$

3.3 Estimation of the term C

The following proposition gives the estimation of the last term.

Proposition 4 *Under the hypothesis of Proposition 2, it holds that, almost surely*

$$\sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)}\sum_k\int_0^1 Y_{j,k}^{2p-2}(1)K_{j,k}^2(\cdot,s)u_s^2 ds<\infty \tag{19}$$

Proof: We will proof (19) by induction on p. For $p=1$, relation (19) becomes

$$\sup_{j\geq 1} 2^{-2j(1-H))}\sum_k\int_0^1 K_{j,k}^2(\cdot,s)u_s^2 ds<\infty \tag{20}$$

But, it is clear from (8) that

$$\sup_{j\geq 1} 2^{-2j(1-H))} \int_0^1 K_{j,k}^2(\cdot,s)u_s^2 ds$$

$$\leq c(H)\|u\|_\infty^2 \sup_{j\geq 1} 2^{-2j(1-H))} 2^{j(1-2H)} 2^j \leq \infty$$

Suppose the inequality true for $p-1$ and let us prove it at the step p.

Writing again the property (3) , we will have

$$\begin{aligned} Y_{j,k}^{2p-2}(1) &= \int_0^1 Y_{j,k}^{2p-3}(1) u_r K_{j,k}(\cdot,r) dW_r \\ &+ (2p-3)\int_0^1 Y_{j,k}^{2p-4}(1) u_r^2 K_{j,k}^2(\cdot,r) dr \\ &+ (2p-3)\int_0^1 Y_{j,k}^{2p-4}(1) u_r K_{j,k}(\cdot,r) v_{j,k}(r) dr \end{aligned}$$

Thus, we can write the expression (19) as the sum of three term. The first is similar with the one denoted by A and it can be estimated analogously. The last one can be estimated as the the term denoted B, and for the second term we can apply the induction hypothesis, because

$$\sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} \sum_k \int_0^1 \left(\int_0^1 Y_{j,k}^{2p-4}(1) K_{j,k}^2(\cdot,r) dr \right) K_{j,k}^2(\cdot,s) u_s^2 ds$$

$$\leq \sup_{j\geq 1} 2^{-2jp\left(\frac{1}{2}-H+\frac{1}{2p}\right)} 2^{j(1-2H)} \sum_k \int_0^1 u_r^2 Y_{j,k}^{2p-4}(1) K_{j,k}^2(\cdot,r) dr$$

and this is finite by the induction hypothesis. □ Now, we give the main result

Theorem 2 *Let $u \in L^\infty([0,1]\times\Omega) \cap L^{2,2}$. Moreover, suppose that*

$$\sup_{\omega,r,\alpha} (D_\alpha u_r)^2 < M \text{ and } \sup_{\omega,r,\alpha} \int_0^1 (D_\beta D_\alpha u_r)^2 d\beta < M \tag{21}$$

for some constant $M > 0$. Moreover assume that the process u is Hölder continuous of order $1-H$ in the Sobolev norm $\|\cdot\|_{1,2}$. Then the paths of the process $X_t = \int_0^t u_s dB_s^H$ belongs almost surely to the Besov space $\mathcal{B}_{p,\infty}^H$ for any $p > \frac{1}{H}$.

Proof: Recall that,

$$\begin{aligned} X_t &= \int_0^t u_s dB_s^H = \int_0^t u_s K(t,s) dW_s \\ &+ \int_0^t \left(\int_s^t (u_r - u_s) \frac{\partial K}{\partial r}(r,s) dr \right) dW_s = Y_t + Z_t \end{aligned}$$

The regularity of the process Y is the consequence of Section 3. For the second integral Z, we have, using anticipating Fubini theorem

$$Z_t = \int_0^t \left(\int_0^r (u_r - u_s) \frac{\partial K}{\partial r}(r,s) dW_s \right) dr$$

and this term is Hölder of order $\frac{1}{2}$ if $v_r = \int_0^r (u_r - u_s) \frac{\partial K}{\partial r}(r,s) dW_s$ belongs to $L^2(\Omega)$. Since the quantity $\frac{\partial K}{\partial r}(r,s)$ behaves as $(r-s)^{H-\frac{3}{2}}$, the Hölder continuity of u in the norm $\| \cdot \|_{1,2}$ implies the conclusion. □

Remark 1: In the case when the process u is adapted (or, more generally, if it belongs to the space L^F, see [3] for the definition), one can obtain the $\frac{1}{2}$-Hölder continuity of the process

$$Z_t = \int_0^t \left(\int_0^r (u_r - u_s) \frac{\partial K}{\partial r}(r,s) dW_s \right) dr$$

under different conditions. Suppose that

$$h_1 = \int_0^1 \int_0^r E \left| u_r - u_s \right|^2 (r-s)^{2H-3} ds dr < \infty$$

$$h_2 = \int_0^1 \int_0^r \int_s^r E \left| D_\alpha u_r \right|^2 (r-s)^{2H-3} d\alpha ds dr < \infty$$

and

$$h_3 = \int_0^1 \int_0^r \int_0^r \int_s^r E \left| D_\beta D_\alpha u_r \right|^2 (r-s)^{2H-3} d\beta d\alpha ds dr < \infty.$$

Then we have the estimate (see [3] or the proof of Theorem 11 in [1])

$$\int_0^t E \left| \int_0^r (u_r - u_s) \frac{\partial K}{\partial r}(r,s) dW_s \right|^2 dr \leq 2(h_1 + h_2 + h_3)$$

and this is finite from above hypothesis.

Remark 2: Our main result Theorem 2 can be applied also to the regular case $H > \frac{1}{2}$, but in this case the conditions considered on the integrand u are stronger than in [9].

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