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Existence and Nonexistence Results for Reaction-Diffusion Equations in Product of Cones

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Abstract

Problems of existence and nonexistence of global nontrivial solutions to quasilinear evolution differential inequalities in a product of cones are investigated. The proofs of the nonexistence results are based on the test-function method developed, for the case of the whole space, by Mitidieri, Pohozaev, Tesei, Véron. The existence result is established using the method of supersolutions.

Keywords: *nonexistence, blow-up, evolution differential inequalities, cone*

AMS subject classification: Primary 35G25; Secondary 35R45, 35K55, 35L70

1 Introduction

The paper is devoted to condition for the nonexistence of global nontrivial solutions of semilinear differential inequalities of parabolic type in a product of conical domains. Such a formulation implies that the domain is unbounded and the corresponding problem has a nontrivial local solution. Complications occur when one attempts to extend this solution to a global one, that is, to find a solution of the Cauchy problem that is defined in the entire domain under consideration. Here, even for semilinear problem of the form

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u = u^q & \text{in } \mathbb{R}^N \times (0,\infty), \\ u(x,0) = u_0(x) \geq 0 & \text{in } \mathbb{R}^N \end{cases}$$

there existes a so-called critical nonlinearity exponent (equal in the present case to the Fujita $q^* = 1 + 2/N$) such that for $1 < q \leq q^*$ no local (in t) solution can be extended to a global one (by an 'extension' we mean here, of course, one keeping the solution in some local function space). Results of this sort can be formulated also as theorems on the nonexistence of global solutions. Surprising here is the fact that we make no assumptions about the growth of the global solution at infinity. For a more detailed setting of the problem and a survey of the literature see [22].

In place of the above equation one can consider the following differential inequality without initial conditions in the local space of functions with continuous second derivatives:

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta u \geq u^q & \text{in } \mathbb{R}^N \times (0,\infty), \\ u(x,0) \geq 0 & \text{in } \mathbb{R}^N. \end{cases}$$

A critical exponent also arises here, which is equal to the similar-type exponent for equations. An interesting feature of inequalities is the fact that it is sometimes possible to construct explicitly solutions for a supercritical nonlinearity, easily establishing in this way the definitive character of the results obtained.

In the present paper we consider the differential inequality

$$\frac{\partial^k u}{\partial t^k} - \Delta\left(|u|^{m-1}u\right) \geq |u|^q \quad \text{in} \quad K_1 \times K_2 \times (0, +\infty),$$

(and its various generalizations) in the case when in place of the entire space $\mathbb{R}^N$ one considers a product of cones K_1 and K_2, $m \geq 1$, and k is a positive integer. Then the critical exponent depends on a certain characteristics of the cones expressible in terms of the first eigenvalue of the corresponding elliptic problem on the unit sphere. The first results for this problem (with $k = 1$, only for positive solutions) were obtained in [23]. In the present paper the initial and boundary data may change sign but have to verify some integral nonnegativity.

The theory of linear elliptic boundary value problems in a cone goes back to Kondratiev. The nonexistence of solutions of the corresponding semilinear and nonlinear elliptic and parabolic problems is mainly studied by means of a reduction to an integral equation, using results similar to comparison theorems and the maximum principle. For the case of parabolic equations we point out the papers [2, 16] and the already classical book [29]. The state of the art is reflected in the surveys [14] and [4].

In the present paper we prove nonexistence of solutions using the test-function method and do not use comparison principles which are characteristic for the theory of second-order equations. The method enables us to demonstrate virtually immediately, using the techniques of Mitidieri-Pohozaev [22], the nonexistence of solutions for the critical nonlinearity exponent (which previously encountered serious difficulties) and to consider systems of differential inequalities and other classes of problems for which the maximum principle does not hold. The central problem is now the choice of a test function and the estimate of the corresponding integrals involving this function. The nonexistence of solution for the single cone using this technique has been investigated in [8, 10, 12].

In order to show that our result is sharp in the parabolic case ($m = k = 1$), we show that the problem

$$\frac{\partial u}{\partial t} - \Delta u = |u|^q \quad \text{in} \quad K_1 \times K_2 \times (0, \infty),$$

admits global nonnegative solutions for q great that the critical exponent. The proof of this result is based on the method of supersolutions.

2 Notations and preliminary results

Let $n \geq 1$ be an integer, $N_i \geq 3$, $i \in \{1, 2, ..., n\}$, and $N = \sum_{i=1}^n N_i$. The polar coordinates in $\mathbb{R}^{N_i}$ will be denoted by (r_i, ω_i). Let $S^{N_i - 1}$ the unit sphere in $\mathbb{R}^{N_i}$ and Ω_i a domain of S^{N_i-1} with sufficiently smooth boundary $\partial\Omega_i$. We will denote by K_i the cone

$$K_i = \{x_i = (r_i, \omega_i) \in \mathbb{R}^{N_i};\ r_i > 0 \text{ and } \omega_i \in \Omega_i\}$$

and $K = K_1 \times K_2 \times ... \times K_n$. The boundary of K_i (resp. K) is designed by ∂K_i (resp. ∂K). The outward normal vector to $\partial\Omega_i$ (resp. ∂K) is denoted by ν_i (resp. ν).

Let $\lambda_i > 0$ be the smallest Dirichlet eigenvalue for the Laplace-Beltrami operator on Ω_i and Φ_i the associated eigenfunction such that $0 < \Phi_i \leq 1$, and the function

$$\Phi \,:\, (\omega_1, \omega_2, ..., \omega_n) \longmapsto \prod_{i=1}^{n} \Phi_i(\omega_i).$$

Let us introduce also the functions $\Phi^{(i)} = \prod_{j=1, j\neq i}^{n} \Phi_j(\omega_j)$.

Throughout this paper, the letter C denotes a constant which may vary from line to line but is independent of the terms which will take part in any limit process. For any real number $q > 1$, we define the real q' such that $1/q + 1/q' = 1$. We will use the notations

$$\Omega = \Omega_1 \times \Omega_2 \times ... \times \Omega_n,$$

whose elements are denoted by $\omega = (\omega_1, \omega_2, ..., \omega_n)$, and finally $d\omega = d\omega_1\, d\omega_2 \ldots d\omega_n$.

We shall construct the test function which will be used in our proofs. Let $\zeta \in C_0^\infty(\mathbb{R}^+)$ be the standard *cut-off function*

$$\zeta(y) = \begin{cases} 1 & \text{if} \quad 0 \leq y \leq 1, \\ 0 & \text{if} \quad y \geq 2 \end{cases} \quad \text{and} \quad \forall y \in \mathbb{R}, \;\; 0 \leq \zeta(y) \leq 1.$$

Let $p_0 \geq k+1$ and η the function defined by

$$\eta(y) = [\zeta(y)]^{p_0}.$$

Explicit computation shows that there is a positive constant $C(\eta) > 0$ such that, for any $y \geq 0$ and any p, $1 < p \leq p_0$, the estimates

$$\left| \frac{d^i \eta}{dy^i}(y) \right|^p \leq C(\eta) \eta^{p-1}(y), \quad 1 \leq i \leq k, \tag{1}$$

hold true.

Consider the function $t \mapsto \eta(t/\rho^\theta)$, where ρ and θ are positive parameters. We have

$$\operatorname{supp} \left| \eta(t/\rho^\theta) \right| = \{ t \in \mathbb{R}^+, \;\; 0 \leq t \leq 2\rho^\theta \}$$

and

$$\operatorname{supp} \left| \frac{d^k \eta}{dt^k}(t/\rho^\theta) \right| = \{ t \in \mathbb{R}^+, \;\; \rho^\theta \leq t \leq 2\rho^\theta \},$$

where "supp" denotes the support. It follows that

$$\int_{\operatorname{supp} \left| \frac{d^k \eta}{dt^k}(t/\rho^\theta) \right|} \frac{\left| \frac{d^k \eta}{dt^k}(t/\rho^\theta) \right|^p}{\eta^{p-1}(t/\rho^\theta)}\, dt \;\leq\; c_\eta \rho^{-\theta(kp-1)}. \tag{2}$$

Now, consider the functions

$$\xi_1(r_1, .., r_n) = \prod_{i=1}^{n} r_i^{s_i}, \;\; \xi_2(r_1, .., r_n) = \prod_{i=1}^{n} \eta(r_i/\rho), \;\; \text{and} \;\; \xi = \xi_1 \times \xi_2,$$

where $s_i > 0$, $i \in \{1, 2, ..., n\}$. Let us set, for any i in $\{1, 2, ..., n\}$,

$$\xi_1^{(i)}(r_1, ..., r_n) = \prod_{j=1, j\neq i}^{n} r_j^{s_j} \quad \text{and} \quad \xi_2^{(i)}(r_1, ..., r_n) = \prod_{j=1, j\neq i}^{n} \eta(r_j/\rho).$$

We shall give estimates concerning $\partial\xi/\partial r_i$ and $\partial^2\xi/\partial r_i^2$. Since

$$\frac{\partial \xi}{\partial r_i} = s_i r_i^{s_i-1}\xi_1^{(i)}\xi_2 + \frac{1}{\rho}\eta'(r_i/\rho)\xi_1\xi_2^{(i)},$$

there is a constant $c_p > 0$ such that

$$\left|\frac{\partial \xi}{\partial r_i}\right|^p \leq c_p s_i^p r_i^{p(s_i-1)} \left[\xi_1^{(i)}\right]^p \xi_2^p + \frac{c_p}{\rho^p}|\eta'(r_i/\rho)|^p \xi_1^p \left[\xi_2^{(i)}\right]^p.$$

Then, there exists $C > 0$, independent of ρ and r_j, $j \in \{1, 2, ..., n\}$, such that

$$\left|\frac{\partial \xi}{\partial r_i}\right|^p \leq C r_i^{p(s_i-1)} \left[\xi_1^{(i)}\right]^p \xi_2^{p-1}\left(1 + \frac{r_i^p}{\rho^p}\right). \tag{3}$$

Similarly,

$$\begin{aligned} \frac{\partial^2 \xi}{\partial r_i^2} &= s_i(s_i-1)r_i^{s_i-2}\xi_1^{(i)}\xi_2 + \frac{2s_i}{\rho}r_i^{s_i-1}\eta'(r_i/\rho)\xi_1^{(i)}\xi_2^{(i)} + \frac{1}{\rho^2}\eta''(r_i/\rho)\xi_1\xi_2^{(i)} \\ &= r_i^{s_i-2}\left\{s_i(s_i-1)\xi_1^{(i)}\xi_2 + 2s_i\frac{r_i}{\rho}\eta'(r_i/\rho)\xi_1^{(i)}\xi_2^{(i)} + \frac{r_i^2}{\rho^2}\eta''(r_i/\rho)\xi_1^{(i)}\xi_2^{(i)}\right\}. \end{aligned}$$

There exists $C > 0$, independent of ρ and r_j, $j \in \{1, 2, ..., n\}$, such that

$$\left|\frac{\partial^2 \xi}{\partial r_i^2}\right|^p \leq C r_i^{p(s_i-2)} \left[\xi_1^{(i)}\right]^p \xi_2^{p-1}\left(1 + \frac{r_i^p}{\rho^p} + \frac{r_i^{2p}}{\rho^{2p}}\right). \tag{4}$$

On the other hand, we have

$$\Delta(\xi_1\Phi) = \sum_{i=1}^{n}\Delta_i(\xi_1\Phi) = \sum_{i=1}^{n}\left\{r_i^{s_i-2}\left[s_i(s_i-1) + s_i(N_i-1) - \lambda_i\right]\xi_1^{(i)}\right\}\Phi.$$

Let

$$s_i^* = -\frac{N_i-2}{2} + \sqrt{\left(\frac{N_i-2}{2}\right)^2 + \lambda_i}$$

be the positive root of $s_i(s_i-1) + s_i(N_i-1) - \lambda_i = 0$ and

$$\xi_*(r_1, r_2, ..., r_n) = \prod_{i=1}^{n} r_i^{s_i^*}.$$

Then, one has

$$\Delta(\xi_*\Phi) = 0.$$

Finally, we introduce the test function (independent of t)

$$\psi_\rho(x) = \xi_*(r_1, r_2, ..., r_n)\,\xi_2(r_1, r_2, ..., r_n)\,\Phi(\omega_1, \omega_2, ..., \omega_n).$$

If ν_i denotes the outward normal vector to Ω_i, then

$$\frac{\partial \psi_\rho}{\partial \nu_i} = \xi_* \xi_2 \Phi^{(i)} \frac{\partial \Phi_i(\omega_i)}{\partial \nu_i}.$$

The Hopf lemma implies that

$$\frac{\partial \Phi_i(\omega_i)}{\partial \nu_i} \le 0, \quad \text{and} \quad \frac{\partial \psi_\rho}{\partial \nu_i} \le 0.$$

Consequently, we conclude that

$$\frac{\partial \psi_\rho}{\partial \nu}|_{\partial K} \le 0. \tag{5}$$

Let us set Δ_i the Laplacian operator with respect to the variable $x_i = (r_i, \omega_i) \in \mathbb{R}^{N_i}$, $i \in \{1, 2, ..., n\}$, then

$$\begin{aligned}
\Delta(\psi_\rho)(x) &= \sum_{i=1}^{n} \Delta_i(\psi_\rho)(x) \\
&= \sum_{i=1}^{n} \left\{ \frac{\partial^2(\psi_\rho)}{\partial r_i^2}(x) + \frac{N_i - 1}{r_i} \frac{\partial(\psi_\rho)}{\partial r_i}(x) + \frac{1}{r_i^2} \Delta_{\omega_i}(\psi_\rho)(x) \right\} \\
&= \sum_{i=1}^{n} \left\{ \frac{\partial^2}{\partial r_i^2} + \frac{N_i - 1}{r_i} \frac{\partial}{\partial r_i} - \frac{\lambda_i}{r_i^2} \right\} \psi_\rho(x) \\
&= \Phi(\omega_1, .., \omega_n) \sum_{i=1}^{n} \left\{ \frac{\partial^2}{\partial r_i^2} + \frac{N_i - 1}{r_i} \frac{\partial}{\partial r_i} - \frac{\lambda_i}{r_i^2} \right\} (\xi_* \xi_2)(r_1, .., r_n),
\end{aligned}$$

since $\Delta_{\omega_i}(\psi_\rho)(x) = -\lambda_i(\psi_\rho)(x)$. Hence,

$$\begin{aligned}
|\Delta_i(\psi_\rho)|^p &= \Phi^p \left| \left\{ \frac{\partial^2}{\partial r_i^2} + \frac{N_i - 1}{r_i} \frac{\partial}{\partial r_i} - \frac{\lambda_i}{r_i^2} \right\} (\xi_* \xi_2) \right|^p \\
&\le C \Phi^p r_i^{p(s_i - 2)} \left[\xi_*^{(i)} \right]^p \xi_2^{p-1} \left(1 + \frac{r_i^p}{\rho^p} + \frac{r_i^{2p}}{\rho^{2p}} \right) \\
&\le C \psi_\rho^{p-1} \frac{\xi_*}{r_i^{2p}} \left(1 + \frac{r_i^p}{\rho^p} + \frac{r_i^{2p}}{\rho^{2p}} \right).
\end{aligned}$$

Since $\eta(r_i/\rho) = 1$ for $r_i \le \rho$ and $\eta(r_i/\rho) = 0$ for $r_i \ge 2\rho$, if we set $\mathcal{N}_i = \{x \in K;\ \Delta_i \psi_\rho \ne 0\}$, then $\mathcal{N}_i \subset \{x \in K;\ \rho < r_i < 2\rho\}$, and the expression

$$1 + \frac{r_i^p}{\rho^p} + \frac{r_i^{2p}}{\rho^{2p}}$$

is bounded for any $x \in \mathcal{N}_i$. Consequently, there is a constant $C > 0$ such that

$$\forall x \in \mathcal{N}_i,\ \ |\Delta_i(\psi_\rho)(x)|^p \le C \psi_\rho^{p-1}(x) \frac{\xi_*}{\rho^{2p}}. \tag{6}$$

Recall, that $\Delta(\psi_\rho) = \sum_{i=1}^{n} \Delta_i(\psi_\rho)$. Then from (6) we obtain

$$|\Delta(\psi_\rho)|^p \le c \sum_{i=1}^{n} |\Delta_i(\psi_\rho)|^p \le c C \psi_\rho^{p-1}(x) \frac{\xi_*}{\rho^{2p}}.$$

Futhermore,

$$\int_K \frac{|\Delta(\psi_\rho)|^p}{\prod_{j=1}^n |x_j|^{(p-1)\sigma_j}\ \psi_\rho^{p-1}}\,dx\ \le$$

$$\sum_{i=1}^n \frac{C}{\rho^{2p}}\int_\rho^{2\rho}\int_{\Omega_i}\left\{\int_{[0,2\rho]^{n-1}}\int_{\prod_{l\neq i}K_{\omega_j}}\prod_{j=1}^n r_j^{s_j^*+N_j-1-(p-1)\sigma_j}\prod_{l\neq i}dr_l\,d\omega_j\right\}d\omega_i\,dr_i.$$

Assume that

$$s_j^*+N_j-(p-1)\sigma_j>0,\ \ 1\le j\le n,$$

which is equivalent to

$$\sigma_j<\frac{s_j^*+N_j}{p-1},\ \ 1\le j\le n, \tag{7}$$

then we have the estimate

$$\int_K \frac{|\Delta(\psi_\rho)|^p}{\prod_{j=1}^n |x_j|^{(p-1)\sigma_j}\ \psi_\rho^{p-1}}\,dx\ \le C\,\rho^{-2p+\sum_{j=1}^n(s_j^*+N_j-(p-1)\sigma_j)}. \tag{8}$$

Finally, we consider the test function, which depends on the all variables,

$$\varphi_\rho(x,t)=\eta\left(\frac{t}{\rho^\theta}\right)\psi_\rho(x). \tag{9}$$

Using (7) and (8), we obtain the first estimate concerning φ_ρ:

$$\begin{aligned}
&\int_0^{+\infty}\int_K \frac{|\Delta(\varphi_\rho)|^p}{\prod_{j=1}^n |x_j|^{(p-1)\sigma_j}\ \varphi_\rho^{p-1}}(x,t)\,dx\,dt\\
\le\ &\int_0^{2\rho^\theta}\eta\left(\frac{t}{\rho^\theta}\right)dt\int_K \frac{|\Delta(\psi_\rho)|^p}{\prod_{j=1}^n |x_j|^{(p-1)\sigma_j}\ \psi_\rho^{p-1}}\,dx\\
\le\ &C\,\rho^{\theta-2p+\sum_{j=1}^n(s_j^*+N_j-(p-1)\sigma_j)}.
\end{aligned} \tag{10}$$

Similarly, using (7) and (2), we obtain the second estimate concerning φ_ρ:

$$\begin{aligned}
&\int\int_{\mathrm{supp}\left|\frac{\partial^k\varphi_\rho}{\partial t^k}\right|}\frac{\left|\frac{\partial^k\varphi_\rho}{\partial t^k}(x,t)\right|^p}{\prod_{j=1}^n |x_j|^{(p-1)\sigma_j}\ \varphi_\rho^{p-1}(x,t)}\,dx\,dt\\
\le\ &\int_K \frac{\psi_\rho(x)}{\prod_{j=1}^n |x_j|^{(p-1)\sigma_j}}\,dx\int_{\mathrm{supp}\left|\frac{d^k\eta}{dt^k}(t/\rho^\theta)\right|}\frac{\left|\frac{d^k\eta}{dt^k}(t/\rho^\theta)\right|^p}{\eta^{p-1}(t/\rho^\theta)}\,dt\\
\le\ &C\,\rho^{\sum_{j=1}^n(s_j^*+N_j)-(p-1)\sigma_j}\rho^{-\theta(kp-1)}\\
\le\ &C\,\rho^{-\theta(kp-1)+\sum_{j=1}^n(s_j^*+N_j-(p-1)\sigma_j)}.
\end{aligned} \tag{11}$$

3 Nonexistence Results

Let us consider the nonexistence problem for weak solutions to the problem

$$(E)\quad \begin{cases} \dfrac{\partial^k u}{\partial t^k} - \Delta\left(|u|^{m-1}u\right) \geq |u|^q \prod_{i=1}^{n} |x_i|^{\sigma_i}, \quad x \in K, \ \ t > 0, \\[2ex] \liminf\limits_{\substack{R_i \to +\infty \\ 1 \leq i \leq n}} \displaystyle\int_{|x_1|<R_1} \cdots \int_{|x_n|<R_n} \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\, \Psi(x)\, dx \geq 0, \quad x \in K, \\[2ex] \liminf\limits_{\substack{R_i \to +\infty \\ 0 \leq i \leq n}} \displaystyle\int_0^{R_0} \int_{|x_1|<R_1} \cdots \int_{|x_n|<R_n} |u(x,t)|^{m-1} u(x,t) \frac{\partial \Psi(x)}{\partial \nu}\, dx\, dt \leq 0, \\ \hfill x \in \partial K, \end{cases}$$

where $\sigma_i \geq 0$ $(i = 1, \ldots, n)$,

$$\Psi(x) = \Psi(r_1, ..., r_n, \omega_1, ..., \omega_n) = \xi_*(r_1, ..., r_n)\Phi(\omega_1, ..., \omega_n),$$

for any $x \in K$.

Definition 1 *A weak solution u of the system (E) on $K\times]0, +\infty[$ is continuous function on $\overline{K} \times [0, +\infty[$ such that the traces $\frac{\partial^j u}{\partial t^j}(x, 0)$, $j \in \{1, .., k-1\}$, are well defined and locally integrable on K and the inequality*

$$\int_0^\infty \int_K \left(|u|^{m-1}\, u\, \Delta\varphi - u\,(-1)^k \frac{\partial^k \varphi}{\partial t^k} + \varphi |u|^q \prod_{i=1}^{n} |x_i|^{\sigma_i} \right) dx\, dt -$$

$$\int_0^\infty \int_{\partial K} |u|^{m-1} u\, \frac{\partial \varphi}{\partial \nu}\, dx\, dt + \sum_{j=0}^{k-1} (-1)^j \int_K \frac{\partial^{k-1-j} u}{\partial t^{k-1-j}}(x,0) \frac{\partial^j \varphi}{\partial t^j}(x,0)\, dx \leq 0, \qquad (12)$$

holds true, for any nonnegative test function $\varphi \in C^{2,k}(K\times]0, +\infty[)$ with compact support such that $\varphi|_{\partial K\times]0,+\infty[} = 0$.

Theorem 1 *Assume that*

$$0 \leq \sigma_j < \frac{q-m}{m}\left(s_j^* + N_j\right), \quad 1 \leq j \leq n$$

and

$$1 < q \leq q^*(k, m) = m + \frac{\left(2 + \sum_{i=1}^n \sigma_i\right)(1 + (k-1)m)}{k\left(-2 + \sum_{j=1}^n \left(s_j^* + N_j\right)\right) + 2}.$$

Then the problem (E) has no nontrivial global weak solution.

We start by proving the following lemmas

Lemma 1 *The assumption*

$$\liminf_{\substack{R_i \to +\infty \\ 1 \leq i \leq n}} \int_{|x_1|<R_1} \cdots \int_{|x_n|<R_n} \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\, \Psi(x)\, dx \geq 0$$

implies that

$$\liminf_{\rho \to +\infty} \int_K \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0) \psi_\rho(x)\, dx \geq 0.$$

Proof 1 *Let us set*

$$v(r_1, r_2, ..., r_n) = \int_\Omega \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\Psi(x)\, d\omega.$$

Then, we have

$$\int_K \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\psi_\rho(x)\, dx = \int_{[0,2\rho]^n} v(r_1, r_2, ..., r_n) \prod_{j=1}^{n} \eta\left(\frac{r_j}{\rho}\right)\, dr_1\, dr_2 \ldots dr_n.$$

Using an integration by parts with respect to the variable r_1 and the fact that $\eta(r_1/\rho) = 1$ for $0 \le r_1 \le \rho$ and $\eta(r_1/\rho) = 0$ for $r_2 \ge 2\rho$, we have

$$\begin{aligned}
\int_0^{2\rho} v(r_1, r_2, ..., r_n)\eta\left(\frac{r_1}{\rho}\right)\, dr_1 &= -\frac{1}{\rho}\int_0^{2\rho}\left(\int_0^{r_1} v(s_1, r_2, ..., r_n)\, ds_1\right)\eta'\left(\frac{r_1}{\rho}\right)\, dr_1 \\
&= -\frac{1}{\rho}\int_\rho^{2\rho}\left(\int_0^{r_1} v(s_1, r_2, ..., r_n)\, ds_1\right)\eta'\left(\frac{r_1}{\rho}\right)\, dr_1 \\
&= -\eta'\left(\frac{\rho_1}{\rho}\right)\int_0^{\rho_1} v(s_1, r_2, ..., r_n)\, ds_1,
\end{aligned}$$

where ρ_1 is such that $\rho \le \rho_1 \le 2\rho$, according to the intermediate value theorem. Proceeding in the same manner for the other variables, r_2, r_3,..., r_n, we obtain

$$\int_K \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\psi_\rho(x)\, dx = (-1)^n \prod_{j=1}^{n} \eta'\left(\frac{\rho_j}{\rho}\right)\int_0^{\rho_1} \ldots \int_0^{\rho_n} v(s_1, ..., s_n)\, ds_1 \ldots ds_n,$$

where ρ_j are such that $\rho \le \rho_j \le 2\rho$, for $j \in \{1, 2, ..., n\}$. Since the cut-off function η is decreasing on $[1, 2]$ then

$$(-1)^n \prod_{j=1}^{n} \eta'\left(\frac{\rho_j}{\rho}\right) \ge 0.$$

On the other hand,

$$\int_0^{\rho_1} \ldots \int_0^{\rho_n} v(s_1, ..., s_n)\, ds_n \ldots ds_1 = \int_{D_\rho} \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\Psi(x)\, dx,$$

where $D_\rho = \{x = (x_1, ..., x_n) \in K;\ |x_j| < \rho_j,\ for\ 1 \le j \le n\}$. Hence, the assumption

$$\liminf_{\substack{R_i \to +\infty \\ 1 \le i \le n}} \int_{|x_1| < R_1} \ldots \int_{|x_n| < R_n} \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\, \Psi(x)\, dx \ge 0,$$

implies that

$$\liminf_{\rho \to +\infty} \int_{D_\rho} \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\Psi(x)\, dx \ge 0.$$

Finally, since the expressions

$$\int_{D_\rho} \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\Psi(x)\, dx \quad and \quad \int_K \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\psi_\rho(x)\, dx$$

have the same sign, it follows that

$$\liminf_{\rho \to +\infty} \int_K \frac{\partial^{k-1} u}{\partial t^{k-1}}(x,0)\psi_\rho(x)\, dx \ge 0,$$

which achieves the proof.

Lemma 2 *The assumption*

$$\liminf_{\substack{R_i\to+\infty\\ 0\le i\le n}} \int_0^{R_0}\int_{|x_1|<R_1}\dots\int_{|x_n|<R_n} |u(x,t)|^{m-1}u(x,t)\frac{\partial\Psi(x)}{\partial\nu}\,dx\,dt\le 0,\quad x\in\partial K$$

implies that

$$\liminf_{\rho\to+\infty}\int_0^{+\infty}\int_{\partial K}|u|^{m-1}u\frac{\partial\varphi_\rho}{\partial\nu}\,dx\,dt\le 0.$$

Proof 2 *The same arguments used in the last proof give the result.*

Now, we are able to give the proof of Theorem 1:

Proof 3 *Assume that the problem (E) admits a global weak solution u. In definition 1, we choose the test function $\varphi(x,t)=\varphi_\rho(x,t)$, defined in (9). Note that φ_ρ satisfies the equalities*

$$\frac{\partial^j\varphi_\rho}{\partial t^j}(x,0)=0,\quad \textit{for}\quad j\in\{1,2,...,k-1\}.$$

Then, the inequality (12) implies that

$$\int_0^\infty\int_K\varphi_\rho|u|^q\prod_{j=1}^n|x_j|^{\sigma_j}\,dx\,dt\;-\int_0^\infty\int_{\partial K}|u|^{m-1}u\,\frac{\partial\varphi_\rho}{\partial\nu}\,dx\,dt+$$

$$\int_K\frac{\partial^{k-1}u}{\partial t^{k-1}}(x,0)\varphi_\rho(x,0)\,dx\;\le\int_0^\infty\int_K\left(|u|^{m-1}\,u\,(-\Delta\varphi_\rho)+u\,(-1)^k\,\frac{\partial^k\varphi_\rho}{\partial t^k}\right)\,dx\,dt.$$

Let ε be an arbitrary postive real number. Using Lemma 1 and Lemma 2, there is $\rho_0>0$ such that for any $\rho\ge\rho_0$,we have

$$\begin{aligned}&\int_0^\infty\int_K|u|^q\varphi_\rho\prod_{j=1}^n|x_j|^{\sigma_j}\,dx\,dt\\ \le\;&\int_0^\infty\int_K\left(|u|^{m-1}\,u\,(-\Delta\varphi_\rho)+u\,(-1)^k\,\frac{\partial^k\varphi_\rho}{\partial t^k}\right)\,dx\,dt+\varepsilon\\ \le\;&\int_0^\infty\int_K\left(|u|^m|\Delta\varphi_\rho|+|u|\left|\frac{\partial^k\varphi_\rho}{\partial t^k}\right|\right)\,dx\,dt+\varepsilon.\end{aligned}\tag{13}$$

Let us set

$$\begin{cases} I_\rho &=\displaystyle\int_0^\infty\int_K|u|^q\varphi_\rho\prod_{j=1}^n|x_j|^{\sigma_j}\,dx\,dt,\\ \mathcal{A}_\rho &=\displaystyle\int_0^\infty\int_K\frac{|\Delta(\varphi_\rho)|^{(\frac{q}{m})'}}{\left(\prod_{j=1}^n|x_j|^{\sigma_j}\,\varphi_\rho(x,t)\right)^{(\frac{q}{m})'-1}}\,dx\,dt,\\ \mathcal{B}_\rho &=\displaystyle\int_0^\infty\int_K\frac{\left|\frac{\partial^k\varphi_\rho}{\partial t^k}\right|^{q'}}{\left(\prod_{j=1}^n|x_j|^{\sigma_j}\,\varphi_\rho(x,t)\right)^{q'-1}}\,dx\,dt.\end{cases}$$

Applying the Hölder inequality to $\int_0^\infty\int_K|u|^m|\Delta\varphi_\rho|\,dx\,dt$, we have

$$\int_0^\infty\int_K|u|^m|\Delta\varphi_\rho|\,dx\,dt\le I_\rho^{\frac{m}{q}}\,\mathcal{A}_\rho^{\frac{1}{(\frac{q}{m})'}}.$$

Similarly, the estimate

$$\int_0^\infty \int_K |u| \left|\frac{\partial^k \varphi_\rho}{\partial t^k}\right| \, dx\, dt \;\le I_\rho^{\frac{1}{q}} \mathcal{B}_\rho^{\frac{1}{q'}},$$

holds true, and we conclude that

$$I_\rho \le I_\rho^{\frac{m}{q}} \mathcal{A}_\rho^{\frac{1}{(\frac{q}{m})'}} + I_\rho^{\frac{1}{q}} \mathcal{B}_\rho^{\frac{1}{q'}} + \varepsilon. \tag{14}$$

Applying the Young inequality to the right-hand side of (14), there is a positive constant C, independent of ρ, such that

$$I_\rho \le C\;(\mathcal{A}_\rho + \mathcal{B}_\rho) + \varepsilon. \tag{15}$$

Combining the estimates (10), (11) and (15) we conclude, for ρ sufficiently large, that

$$I_\rho \le C\,(\rho^{\alpha_1} + \rho^{\alpha_2}) + \varepsilon, \tag{16}$$

where

$$\begin{aligned}
\alpha_1 &= \theta - 2\left(\frac{q}{m}\right)' + \sum_{j=1}^n \left(s_j^* + N_j - \left(\left(\frac{q}{m}\right)' - 1\right)\sigma_j\right),\\
\alpha_2 &= -\theta(kq' - 1) + \sum_{j=1}^n (s_j^* + N_j - (q' - 1)\sigma_j),
\end{aligned}$$

and C is a positive constant independent of ρ.
At this stage, we choose the parameter θ to equal the exponents of ρ in the last estimate. Explicit computation gives

$$\theta = \frac{1}{k}\left(\frac{q-1}{q-m}\left(2 + \sum_{i=1}^n \sigma_i\right) - \sum_{i=1}^n \sigma_i\right) = \frac{1}{k}\cdot\frac{2(q-1) + (m-1)\sum_{i=1}^n \sigma_i}{q-m}.$$

Since $k \ge 1$ and $m \ge 1$ it follows that $\theta > 0$. Now, the estimate (16) can be rewritten

$$I_\rho \le C\rho^{\frac{\alpha}{k(q-m)}} + \varepsilon,$$

where

$$\alpha = q\left\{k\left(-2 + \sum_{j=1}^n (s_j^* + N_j)\right) + 2\right\} - 2 - (m(k-1)+1)\sum_{j=1}^n \sigma_j - km\sum_{j=1}^n (s_j^* + N_j).$$

Now, we require

$$\frac{\alpha}{k(q-m)} \le 0,$$

which is equivalent to

$$q \le q^*(k,m) = m + \frac{\left(2 + \sum_{i=1}^n \sigma_i\right)(1 + (k-1)m)}{k\left(-2 + \sum_{j=1}^n (s_j^* + N_j)\right) + 2}.$$

Whence, I_ρ is bounded uniformly with respect to the parameter ρ. Moreover, the function $I(\rho)$ is increasing in ρ. Consequently, the monotone convergence theorem implies that the function

$$(x,t) \equiv (r,\omega,t) \longmapsto |u^q(x,t)|\,\Psi(x)\,\prod_{j=1}^{n}|x_j|^{\sigma_j}$$

is in $L^1(K\times]0,+\infty[)$. Furthermore, note that

$$supp(\Delta\varphi_\rho) \subset \{t \in \mathbb{R}^+,\quad 0 \le t \le 2\rho^\theta\} \times \{x \in K,\quad \rho \le |x_j| \le 2\rho,\, 1 \le j \le n\}$$

and

$$supp\left(\frac{\partial^k \varphi_\rho}{\partial t^k}\right) \subset \{t \in \mathbb{R}^+,\quad \rho^\theta \le t \le 2\rho^\theta\} \times \{x \in K,\quad 0 \le |x_j| \le 2\rho,\, 1 \le j \le n\}.$$

Whence, instead of (14) we have more precisely

$$I_\rho \le \tilde{I}_\rho^{\frac{m}{q}}\,\mathcal{A}_\rho^{\frac{1}{(\frac{q}{m})'}} + \tilde{I}_\rho^{\frac{1}{q}}\,\mathcal{B}_\rho^{\frac{1}{q'}} + \varepsilon,$$

where

$$\tilde{I}_\rho = \int_{\mathcal{C}_\rho} \varphi_\rho\,|u|^q \prod_{j=1}^{n}|x_j|^{\sigma_j}\,dx\,dt,$$

and

$$\mathcal{C}_\rho = supp(\Delta\varphi_\rho) \cup supp\left(\frac{\partial^k \varphi_\rho}{\partial t^k}\right).$$

Finally, using the dominated convergence theorem, we obtain that there is $\rho_0' > 0$ such that

$$I_\rho \le \varepsilon, \quad \text{for any} \quad \rho \ge \rho_0',$$

which is equivalent to

$$\lim_{\rho \to +\infty} I_\rho = 0.$$

This means that $u \equiv 0$, which contradicts the fact that u is assumed to be nontrivial weak solution to (E).

4 Existence Results

In this section, we will limit ourselves to the problem

$$(\tilde{\mathrm{E}}) = \begin{cases} \frac{\partial u}{\partial t} - \Delta u = u^q & \text{in} \quad K\times]0,+\infty[, \\ u(x,t) = 0 & \text{on} \quad \partial K \times [0,+\infty[, \\ u(x,0) = u_0(x) \ge 0 & \text{in} \quad K, \end{cases}$$

>From Theorem 1, we know that if

$$1 < q \le q^* = 1 + \frac{2}{\sum_{i=1}^{n}(N_i + s_i^*)}$$

then the problem $(\tilde{\mathrm{E}})$ has no global nontrivial solution. We complete this result by the existence one:

Theorem 2 *If*

$$q > q^* \equiv 1 + \frac{2}{\sum_{i=1}^{n}(N_i + s_i^*)}$$

then nontrivial global solutions of $(\tilde{E})$ *exist.*

Proof 4 *The proof is based on the method of supersolutions [30, 31] and the arguments used are inspired by the ideas of [15, 16, 23].*

Let v *be a positive solution of*

$$\begin{cases} \frac{\partial v}{\partial t} - \Delta v = 0 \quad in \quad K\times]0, +\infty[, \\ v(x,t) = 0 \quad on \quad \partial K \times [0, +\infty[, \\ v(x,0) = v_0(x) \geq 0 \quad in \quad K, \end{cases}$$

and let the function w *defined on* $K\times]0, +\infty[$ *by* $w(x,t) = \alpha(t)v(x,t)$*, where the function* α *has to be defined. If* α *is selected such that*

$$\alpha'(t) = (\alpha(t))^q \left[\sup_{x\in K} v(x,t)\right]^{q-1},$$

then w *is a supersolution of* $(\tilde{E})$ *on its interval of definition. Let us set* α *be the solution of the Cauchy problem*

$$\begin{cases} \alpha'(t) = (\alpha(t))^q ||v(.,t)||_{L^\infty(K)}^{q-1}, \quad t > 0, \\ \alpha(0) = \alpha_0 > 0. \end{cases} \tag{17}$$

It is easy to see that the solution of (17) is global if, and only if,

$$\int_0^{+\infty} ||v(.,t)||_{L^\infty(K)}^{q-1}\, dt < +\infty \tag{18}$$

and

$$0 < \alpha_0 < \left((q-1)\int_0^{+\infty} ||v(.,t)||_{L^\infty(K)}^{q-1}\, dt\right)^{\frac{-1}{q-1}}.$$

At this stage, we will construct the function v *on* $\overline{K} \times [0, +\infty[$*. Consider the function* v_i *defined on* $K_i\times]0, +\infty[$ *by*

$$v_i(x_i,t) = \frac{1}{t+1}\frac{1}{r_i^{\frac{1}{2}(N_i-2)}} I_{\nu_i}\left(\frac{r_i}{2(t+1)}\right)\exp\left(-\frac{r_i^2+1}{4(t+1)}\right)\Phi_i(\omega_i),$$

where $x_i = (r_i, \omega_i)$*,* $\nu_i = s_i^* + \frac{N_i-2}{2}$ *and* I_{ν_i} *is the modified Bessel function of order* ν_i *[32]. The function* v_i *is a positive solution of*

$$\frac{\partial v_i}{\partial t} - \Delta v_i = 0 \ \ in \ \ K_i\times]0, +\infty[$$

(see, for example, [16]). Recall that the asymptotic behaviour of I_{ν_i} *in the neighborhood of* 0 *and* $+\infty$ *is given respectively by [32]*

$$I_{\nu_i}(z) \sim \frac{z^{\nu_i}}{2^{\nu_i}\Gamma(\nu_i+1)} \quad as \ \ z \longrightarrow 0^+ \tag{19}$$

and

$$I_{\nu_i}(z) \sim \frac{\exp(z)}{\sqrt{2\pi z}} \quad as \quad z \longrightarrow +\infty. \tag{20}$$

Then, using the fact that $\Phi_i = 0$ *on* $\partial\Omega_i$ *and (19), we conclude that* v_i *vanishes on* $\partial K_i \times [0, +\infty[$.

Let us set now

$$v(x,t) = \prod_{i=1}^{n} v_i(x_i, t), \quad for\ any\ \ x = (x_1, x_2, ..., x_n) \in K, \ t > 0$$

and

$$V_i(r_i, t) = \frac{1}{t+1} \frac{1}{r_i^{\frac{1}{2}(N_i-2)}} I_{\nu_i}\left(\frac{r_i}{2(t+1)}\right) \exp\left(-\frac{r_i^2+1}{4(t+1)}\right).$$

The function v *is a positive solution of*

$$\frac{\partial v}{\partial t} - \Delta v = 0 \quad in \quad K \times]0, +\infty[,$$

which vanishes on $\partial K \times [0, +\infty[$. *Morover, if the following estimate*

$$\limsup_{t \to +\infty} (t+1)^{\frac{1}{q^*-1}} \left[\sup_{\substack{r_i > 0 \\ 1 \le i \le n}} \prod_{i=1}^{n} V_i(r_i, t) \right] < +\infty \tag{21}$$

holds then the condition (18) will be satisfied for any $q > q^*$. *Indeed, it suffices to remark that* $0 < \Phi_i \le 1$ *for* $1 \le i \le n$, *and*

$$\int_0^{+\infty} (t+1)^{-\frac{q-1}{q^*-1}} \, dt < +\infty, \quad for\ any\ \ q > q^*.$$

We will show now that the esimate (21) is satisfied. First, since

$$\lim_{\substack{r_i \to 0 \\ 1 \le i \le n}} \prod_{i=1}^{n} V_i(r_i, t) = \lim_{\substack{r_i \to \infty \\ 1 \le i \le n}} \prod_{i=1}^{n} V_i(r_i, t) = 0, \quad \forall\, t > 0,$$

then, for any $t > 0$, *there exists* $0 < r_i^*(t) < +\infty$, $1 \le i \le n$, *such that*

$$\prod_{i=1}^{n} V_i(r_i^*(t), t) = \sup_{\substack{r_i > 0 \\ 1 \le i \le n}} \prod_{i=1}^{n} V_i(r_i, t).$$

Let

$$\mathcal{V}(t) = (t+1)^{\frac{1}{q^*-1}} \prod_{i=1}^{n} V_i(r_i^*(t), t).$$

Using the fact that

$$\frac{1}{q^*-1} = \frac{1}{2} \sum_{i=1}^{n} (N_i + s_i^*),$$

we can write

$$\mathcal{V}(t)=\prod_{i=1}^{n}\left\{(t+1)^{\frac{s_i^*}{2}}\left(\frac{r_i^*(t)}{t+1}\right)^{-\frac{N_i-2}{2}} I_{\nu_i}\left(\frac{r_i^*(t)}{2(t+1)}\right)\exp\left(-\frac{(r_i^*(t))^2+1}{4(t+1)}\right)\right\}.$$

If we set

$$y_i^*(t)=\frac{r_i^*(t)}{2(t+1)},$$

then

$$\mathcal{V}(t)=e^{-\frac{n}{4(t+1)}}\prod_{i=1}^{n}\left\{(t+1)^{\frac{s_i^*}{2}}(y_i^*(t))^{-\frac{N_i-2}{2}}I_{\nu_i}(y_i^*(t))e^{-(t+1)(y_i^*(t))^2}\right\}.$$

Let, for $1\leq i\leq n$,

$$\mathcal{V}_i(t)=(t+1)^{\frac{s_i^*}{2}}(y_i^*(t))^{-\frac{N_i-2}{2}}I_{\nu_i}(y_i^*(t))e^{-(t+1)(y_i^*(t))^2}.$$

Suppose that there is a sequence $(t_k)_{k\in\mathbb{N}}\to+\infty$ *such that*

$$\lim_{t_k\to+\infty}\mathcal{V}(t_k)=+\infty.$$

Three cases can arise for each $i\in\{1,2,...,n\}$*:*

- *Case 1: There exists a subsequence, also denoted by* $(t_k)_{k\in\mathbb{N}}$, *such that*

$$\lim_{t_k\to+\infty}(y_i^*(t_k))=+\infty.$$

In this case

$$\mathcal{V}_i(t_k)\sim const\cdot(t_k+1)^{\frac{s_i^*}{2}}(y_i^*(t_k))^{-\frac{N_i-1}{2}}e^{y_i^*(t_k)}e^{-(t_k+1)(y_i^*(t_k))^2}\quad as\ \ t_k\to+\infty,$$

which implies that

$$\lim_{t_k\to\infty}\mathcal{V}_i(t_k)=0.$$

- *Case 2: There exists a subsequence, also denoted by* $(t_k)_{k\in\mathbb{N}}$, *such that*

$$\lim_{t_k\to+\infty}(y_i^*(t_k))=0.$$

In this case

$$\mathcal{V}_i(t_k)\sim const\cdot\left[(t_k+1)(y_i^*(t_k))^2\right]^{\frac{s_i^*}{2}}e^{-(t_k+1)(y_i^*(t_k))^2}\quad as\ \ t_k\to+\infty,$$

which implies that $\mathcal{V}_i(t_k)$ *is bounded, since the function* $z\longmapsto z^{\frac{s_i^*}{2}}e^{-z}$ *is bounded on* $\mathbb{R}^+$.

- *Case 3: There are two constants* A_i *and* B_i *such that the sequence* $(y_i^*(t_k))_{k\in\mathbb{N}}$ *satisfies*

$$0<A_i\leq y_i^*(t_k)\leq B_i<+\infty.$$

In this case, the expression $\mathcal{V}_i(t_k)$ *is clearly bounded.*

Whence, there is no subsequence of $(t_k)_{k\in\mathbb{N}}\to+\infty$ *such that*

$$\lim_{t_k\to+\infty}\mathcal{V}_i(t_k)=+\infty,$$

which imply that there is no sequence $(t_k)_{k\in\mathbb{N}}\to+\infty$ *such that*

$$\lim_{t_k\to+\infty}\mathcal{V}(t_k)=+\infty.$$

This ends the proof.

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