



Prépublications du Département de Mathématiques

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On the monodromy representation of polynomials

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novembre 2002

Classification: 14D05, 14D07, 32S20, 32S35, 32S40, 58K10.

Mots clés:

2002/08

On the monodromy representation of polynomials

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November 25, 2002

Abstract

In this paper, we show an intriguing property of the eigen values of the global monodromy operators associated to a complex polynomial map. AMS Classification : 14D05, 14D07, 32S20, 32S35, 32S40, 58K10.

1 Introduction and notations

Let $f : \mathbb{C}^{n+1} \rightarrow \mathbb{C}$ be a non-constant polynomial map, there exists a minimal finite set : $B_f = \{b_1, \dots, b_m\}$, the bifurcation set of f , such that if we denote by $S = \mathbb{C} \setminus B_f$ and $X = f^{-1}(S)$ then $f : X \rightarrow S$ is a topological fiber bundle. If $s \notin B_f$, the generic fiber $F = f^{-1}(s)$ is a smooth algebraic hypersurface. Let D_i be some small disc such that $D_i \cap B_f = b_i$, we choose a regular value s_0 and we construct a set of m disjoint paths from s_0 to a point of ∂D_i in order to obtain a set of generators of the fundamental group $\pi_1(S, s_0)$ in the following way. A generator γ_i is the concatenation of the path from s_0 to a point of ∂D_i , of ∂D_i itself and of the path going back to s_0 . Let $\rho : \pi_1(S, s_0) \rightarrow \text{Aut}(H^k(F, \mathbb{C}))$ be the monodromy representation of f and $T_{b_i} = \rho(\gamma_i)$. We denote by T_∞ the image of a big loop around all the points of B_f . In the following we will only consider complex coefficient homology and cohomology and so we will denote

$$\begin{aligned} H^k(F) &= H^k(F, \mathbb{C}) \\ H_k(F) &= H_k(F, \mathbb{C}) \end{aligned}$$

We will use the notation

$$H^k(F)^{inv} = \bigcap_{i=1}^m \text{Ker}(T_{b_i} - Id)$$

We will suppose that f is primitive i.e. its generic fiber is irreducible or equivalently it is not of the form $g \circ h$ with $g : \mathbb{C} \rightarrow \mathbb{C}$ and $h : \mathbb{C}^n \rightarrow \mathbb{C}$ polynomial maps and $\deg g > 1$, see [6].

I would like to thank Sabir Gusein-Zade for the ideas he has suggested, which led me to the proof.

2 The result

Deligne proved the monodromy theorem in [4] which implies that the eigenvalues of the operators T_{b_i} are some roots of unity. This result is in fact a local one in the base, we have to study the fibration around a small disc sufficiently close to the singularity b_i . However this result can easily be extended to T_∞ in this way, if we think of the complex plane as the Riemann sphere minus one point (the point at infinity), we see that a big loop around all the points of B_f is homotopic to a small disc around the infinity point.

The aim of this article is to give a property of the eigenvalues of $\rho(\gamma)$ for any loop $\gamma \in \pi_1(S, s_0)$, these are no longer roots of unity in general but we have :

Theorem 1. *With the previous notations, let λ be any eigenvalue of $\rho(\gamma)$ then*

$\frac{1}{\lambda}$ is an eigenvalue of $\rho(\gamma)$ and the multiplicity of λ and $\frac{1}{\lambda}$ in the characteristic polynomial of $\rho(\gamma)$ are the same.

Remark 2. *Let P be the characteristic polynomial of $\rho(\gamma)$, the theorem implies that*

$$x^{\deg P} P\left(\frac{1}{x}\right) = \pm P(x)$$

Example 3. *If $f = x + x^2y^2 + x^2y^3$, we have $B_f = \{0, b = -\frac{27}{16}\}$ the monodromy representation is generated by*

$$T_0 = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & j & 0 & 0 \\ 0 & 0 & j^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad T_b = \begin{pmatrix} 1 & 0 & 0 & 0 \\ j^2 & 1 & 0 & 0 \\ j & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where $j = e^{\frac{2i\pi}{3}}$ (see [1] for the details of this calculus). For instance we have

$$T_0 (T_b)^k = \begin{pmatrix} 1 - k & 1 & 1 & 0 \\ k & j & 0 & 0 \\ k & 0 & j^2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

whose characteristic polynomial is $\chi = (x - 1)^2 (x^2 + (k + 1)x + 1)$ which show that the result is true in this case, and that in general, the eigenvalues of $\rho(\gamma)$ are not roots of unity.

Remark 4. *The previous example leads us to a question. If we are given two matrices A and B whose eigenvalues are roots of unity such that their product also has this property. Do all the elements of the group generated by A and B have the property of the theorem? The answer is no, as shows this counter-example :*

Take

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$$

then

$$AB = \begin{pmatrix} -2 & 2 & -1 \\ 1 & 0 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

whose characteristic polynomial is $(x - 1)(x + 1)^2$. But

$$A^{-1}B = \begin{pmatrix} 6 & -3 & 2 \\ -3 & 2 & -1 \\ 2 & -1 & 1 \end{pmatrix}$$

whose characteristic polynomial is $x^3 - 9x^2 + 6x - 1$.

We see here that the property claimed here is indeed in relation with our geometrical situation.

3 The proof

Lemma 5. *Let m be an isomorphism of \mathbb{C}^p then m is similar to its inverse if and only if there exist a non-degenerate bilinear form ϕ invariant by m , i.e. such that*

$$\forall (x, y) \in (\mathbb{C}^p)^2, \phi(m(x), m(y)) = \phi(x, y)$$

Proof. We choose a basis and let M and B be the matrices of m and ϕ with respect to this base. ϕ is invariant by m means

$${}^tMBM = B$$

this is equivalent to

$${}^tM = BM^{-1}B^{-1}$$

It is well known that a matrix is similar to its transpose, and this ends the proof. \square

Remark 6. *One should think of finding a form on $H^k(F)$, non-degenerate and invariant by the modromy operators (one can also say a non-degenerate "flat" form). As we will see later, the monodromy operators are orientation preserving (proposition 7) so if $k = n$ we have a natural form is given by the cup-product :*

$$\begin{aligned} H^n(F) \times H^n(F) &\longrightarrow \mathbb{C} \\ (c, c') &\longrightarrow c \smile c' \end{aligned}$$

but there is a problem, this form is degenerate in general, and most of all, the monodromy is not trivial on its kernel, an example is given in [1], (remarque 4.4).

We will see that it is quite difficult to find a non-degenerate flat form in general, we will need to use the variations of mixed Hodge structures. Nevertheless, in some particular cases we have an elementary and explicit proof. We will first work these two cases, namely the polynomials which are tame (good at infinity) and the polynomials of two variables.

3.1 The tame case

A tame polynomial $f : \mathbb{C}^{n+1} \rightarrow \mathbb{C}$ is defined in [2]. (In fact, everything in this section is also true in the more general case of M-tame polynomials defined in [9]). Such a polynomial map has a lot of properties, for instance we have

$$H_k(F) = 0 \text{ for } k \neq 0, n$$

Let us consider the intersection form

$$\langle \cdot, \cdot \rangle : H_n(F) \times H_n(F) \longrightarrow H_0(F) \approx \mathbb{C}$$

it is flat because the monodromy operators are orientation preserving (see proposition 7). Note that the last isomorphism explain why we have chosen f primitive.

For us the most important property implied by the tame hypothesis is that the kernel of the intersection form is equal to the kernel of $T_\infty - Id$ (see [5], example 4.3). Moreover in [5] A. Némethi and A. Dimca show that the monodromy representation is determined by T_∞ and a certain decomposition of $H_n(F)$, in particular they prove that $Ker(T_\infty - Id) = H_n(F)^{inv}$.

Let $T = \rho(\gamma)$ be any monodromy operator we have the commutative diagramm

$$\begin{array}{ccccccc} 0 \rightarrow & Ker \langle \cdot, \cdot \rangle & \rightarrow & H_n(F) & \rightarrow & \frac{H_n(F)}{Ker \langle \cdot, \cdot \rangle} & \rightarrow 0 \\ & Id \downarrow & & T \downarrow & & \bar{T} \downarrow & \\ 0 \rightarrow & Ker \langle \cdot, \cdot \rangle & \rightarrow & H_n(F) & \rightarrow & \frac{H_n(F)}{Ker \langle \cdot, \cdot \rangle} & \rightarrow 0 \end{array}$$

and the lemma 5 says that $\bar{T} \approx \bar{T}^{-1}$, we can then easily deduce the property claimed on T .

3.2 The two variables case

In this section we take $f : \mathbb{C}^2 \rightarrow \mathbb{C}$ and we want to study the action on $H^1(F)$. As we mentioned before, the monodromy is not trivial in general on the kernel of the intersection form.

The Poincaré Duality Theorem states that the intersection form (or the cup-product) is non degenerated on a compact oriented manifold. So we will make use of a smooth compactification of F . Let us consider

$$\begin{array}{ccc} \phi : \mathbb{P}^2(\mathbb{C}) & \dashrightarrow & \mathbb{P}^1(\mathbb{C}) \\ (x : y : z) & \longmapsto & (\bar{f} : z^d) \end{array}$$

where \bar{f} is the homogenization of f , and d is the degree of f . By a sequence of blowing-up of the indertemination points of ϕ we define $\pi : \mathbb{X} \rightarrow \mathbb{P}^2(\mathbb{C})$ such that $\tilde{\phi} = \phi \circ \pi$ is regular on \mathbb{X} , and such that the total transforms of the fibers of f are normal crossing divisors. See [8] for a detailed construction. Let \tilde{F} denote the strict transform of F and $A = \tilde{F} \setminus F$. The Gysin short exact sequence give the following commutative diagramm

$$\begin{array}{ccccccc} 0 \rightarrow & H^1(\tilde{F}_t) & \rightarrow & H^1(F) & \rightarrow & \tilde{H}^0(A_t) & \rightarrow 0 \\ & \tilde{T} \downarrow & & T \downarrow & & T^0 \downarrow & \\ 0 \rightarrow & H^1(\tilde{F}_t) & \rightarrow & H^1(F) & \rightarrow & \tilde{H}^0(A_t) & \rightarrow 0 \end{array}$$

where T is any monodromy operator, \tilde{T} and T^0 are the corresponding induced monodromy operators. Now we have that $\tilde{T} \approx \tilde{T}^{-1}$ so we just have to check that $T^0 \approx (T^0)^{-1}$. But A is a finite set of points and T^0 is just a permutation of these points and so ${}^t(T^0) = (T^0)^{-1}$. To complete the proofs of the two preceding cases we will now prove the

Proposition 7. *The monodromy operators are orientation preserving.*

Proof. We consider

$$\begin{array}{ccc} \phi : \mathbb{P}^{n+1}(\mathbb{C}) & \dashrightarrow & \mathbb{P}^1(\mathbb{C}) \\ (x_0 : \cdots : x_n) & \longmapsto & (\tilde{f} : z^d) \end{array}$$

Let $\pi : \mathbb{X} \rightarrow \mathbb{P}^{k+1}(\mathbb{C})$ be a map such that $\tilde{f} = \phi \circ \pi$ is regular on \mathbb{X} , and such that the total transforms of the fibers of f are normal crossing divisors. Let $F_0 = f^{-1}(0)$ be a non-generic fiber and let $F = f^{-1}(t)$ a generic fiber with t sufficiently close to 0, it is clear that we just have to prove that the property claimed is true for the monodromy operator m corresponding to a small loop around 0. If we denote by \tilde{m} the operator on $\tilde{F} = \tilde{f}^{-1}(t)$ extending m , it is enough to prove that \tilde{m} is orientation preserving because the measure of the set $\tilde{F} \setminus F$ is zero. But now, we have the situation a family of algebraic varieties acquiring singularities studied in [3]. In this paper, the action of \tilde{m} is described in locally by analytic formulas (p97), and so, it is orientation preserving. \square

4 The general case

The proof is quite short but it involves much more theory. We will recall the basic definitions (see [10]).

Definition 8. *Let E be a \mathbb{Q} -vector space and let $V = E \otimes_{\mathbb{Q}} \mathbb{C}$. A Hodge structure of weight k on V is the data of a decreasing filtration $G = \{G^p\}$ of V so that we have*

$$V = G^p \oplus \overline{G^{q+1}} \text{ if } p + q = k$$

where the "bar" denotes complex conjugation, equivalently

$$V = \bigoplus_{p+q=k} H^{p,q} \text{ where } H^{p,q} = G^p \cap \overline{G^q}$$

The Weil operator

$$C : V \rightarrow V$$

is given by the direct sum of multiplication by i^{p-q} on $H^{p,q}$.

Definition 9. *Let S be a complex manifold. A variation of Hodge structure of weight k over S , is the collection of data $(\mathbf{V}, \mathcal{G})$ where*

1. \mathbf{V} is a locally constant sheaf of \mathbb{Q} vector spaces on S (we also say local system).
2. $\mathcal{G} = \{G^p\}$ is a decreasing filtration by holomorphic subbundles of the bundle $\mathcal{V} = \mathcal{O}_S \otimes_{\mathbb{Q}} \mathbf{V}$, such that at each $s \in S$, \mathcal{G} induces the Hodge filtration G_s of a Hodge structure of weight k on the fiber V_s of \mathcal{V} .

3. Let ∇ be the flat connexion on \mathcal{V} , we have for each p

$$\nabla \mathcal{G}^p \subset \Omega_S^1 \otimes \mathcal{G}^{p-1}$$

Definition 10. A polarization of a variation of Hodge structure of weight k is a non degenerate flat bilinear form

$$\mathcal{P} : \mathbf{V} \times \mathbf{V} \rightarrow \mathbb{Q}$$

which is $(-1)^k$ symmetric such that the hermitian form on the fiber of $\mathbf{V} \otimes_{\mathbb{Q}} \mathbb{C}$

$$(v, w) \rightarrow \mathcal{P}(C_s v, \bar{w})$$

is positive-definite. A variation of Hodge structure is said to be polarizable if it admits a polarization.

Consequently, the first lemma says that we would be able to conclude if $\mathbf{V} = R^k f_* \mathbb{Q}_X$ was the underlying local system of a polarizable variation of Hodge structure. Indeed the form

$$\begin{aligned} H^k(F, \mathbb{Q}) \times H^k(F, \mathbb{Q}) &\rightarrow \mathbb{Q} \\ (v, w) &\rightarrow \mathcal{P}(C_s v, w) \end{aligned}$$

would then be bilinear, flat and non-degenerate. In fact, if $g : X \rightarrow S$ is a smooth and proper holomorphic map then $R^k f_* \mathbb{Q}_X$ has this good structure, but in our situation we will need a more sophisticated notion.

Definition 11. A variation of mixed Hodge structure on a complex manifold S is a collection of data $(\mathbf{V}, W, \mathcal{G})$, where

1. \mathbf{V} is a local system of \mathbb{Q} vector spaces on S ,
2. $W = \{W_k\}$ is an increasing filtration of \mathbf{V} by local subsystems,
3. $\mathcal{G} = \{\mathcal{G}^p\}$ is a decreasing filtration by holomorphic subbundles of the bundle $\mathcal{V} = \mathcal{O}_S \otimes_{\mathbb{Q}} \mathbf{V}$,
4. for each p , $\nabla \mathcal{G}^p \subset \Omega_S^1 \otimes \mathcal{G}^{p-1}$
5. With \mathcal{W}_k denoting $\mathcal{O}_S \otimes W_k$, $(Gr_k^W \mathbf{V}, \mathcal{G}(\mathcal{W}_k / \mathcal{W}_{k-1}))$ is a variation of Hodge structure of weight k , (in particular on the fiber, (V_s, W_s, G_s) is a mixed Hodge structure).
If these variation of Hodge structure are polarizable then the variation of mixed Hodge structure is called graded-polarizable.

Proof of the main theorem. It is proven in [10] §5 that $R^k f_* \mathbb{Q}_X$ the underlying local system of a graded-polarizable variation of mixed Hodge structure. That means that we have a filtration

$$0 = W_{-1} \subset W_0 \subset W_1 \subset \dots \subset W_{2k} = H^k(F)$$

such that there is a flat non degenerate bilinear form on $\frac{W_i}{W_{i-1}}$. Using the following short exact sequences

$$0 \rightarrow W_{i-1} \rightarrow W_i \rightarrow \frac{W_i}{W_{i-1}} \rightarrow 0$$

we show easily by induction that all the restrictions of the monodromy operators on each W_i have the property claimed and so we prove it for T on $H^k(F)$, which concludes the proof. \square

Let

$$f : X \rightarrow S$$

be a morphism of algebraic varieties such that $\dim X = n + 1$, S is a smooth curve and f is a topologically locally trivial fibration with a connected fibre F . The proof of our main theorem also applies to any representation

$$\rho : \pi_1(S) \rightarrow \text{Aut}(H^k(F))$$

associated to f . The typical example is that of rational functions on \mathbb{P}^{n+1} which includes polynomial functions on \mathbb{C}^{n+1} , see for instance [7].

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