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Abstract

We are interested in the type, the position and the move of singularities formed by solutions of a singulary perturbed differential equation $\varepsilon u' = \Phi(z, u, a, \varepsilon)$ with respect to the parameter a. We show in the case of a Riccati equation that the overstability values are logarithmic singularities of the multivalued function « Indicatrice des pôles »: which for each value of the parameter associates the poles of a specific solution called « distinguished solution ». This means that after surrounding an overstability value, the poles move exchanging their positions.

1 Introduction and general results

Let ε be a positive infinitesimal number and $\mathcal{D} \subset \mathbb{C}$ be a simply connected domain. We consider a Riccati equation of the form

$$\varepsilon u' = \varphi^2(z, \varepsilon) - u^2 + \varepsilon a =: \Phi(z, u, a, \varepsilon) \tag{1}$$

where φ is a standard analytic function in a simply connected domain $\Omega \subset \mathbb{C}^2$, and a is a complex parameter. We assume that for each $z \in \mathcal{D}$, $(z, 0) \in \Omega$.

For a fixed value a_0 $(= 0)$ and ε small, the vector field associated to (1) is structured by two limited analytic curves (where the vector field is limited) called *slow curves* and defined by $\Phi(z, u_0(z), a_0, 0) = 0$ $(u_0(z) := \pm\varphi(z, 0))$.
We suppose that the function

$$f(z) = \frac{\partial \Phi}{\partial u}(z, u_0(z), a_0, 0) \tag{2}$$

has a unique zero z_0, which is simple. We denote

$$F(z) = \int_{z_0}^{z} f(s)ds \tag{3}$$

and we consider the landscape associated the *slow curve* u_0

$$R(z) = \Re(F(z)). \tag{4}$$

The landscape (4) is composed with two mountains and two valleys. We have the following results ([3] , [4])

- To each mountain corresponds a unique solution, called « distinguished solution », which remains close to the corresponding *slow curve* up to infinity, on the mountain and the two valleys.
- The distinguished solution has a line of poles infinitely close to the Stokes lines (the boarding lines of the domain where the solution exists).

In his preprint [4], J.L Callot introduced a multivalued function, called « Indicatrice des pôles »$I(a)$ which, to each value of the parameter a, associates the locus of the poles of the distinguished solution. He observed also that there exist some values of the parameter, « the overstability values », for which the distinguished solution remains close to the slow curve on the second mountain too. He conjectured that these values correspond to logarithmic singularities of $I(a)$. In our work, for convenience $I(a)$ will be the image by F/ε, see (3) and (12) of these singularities.

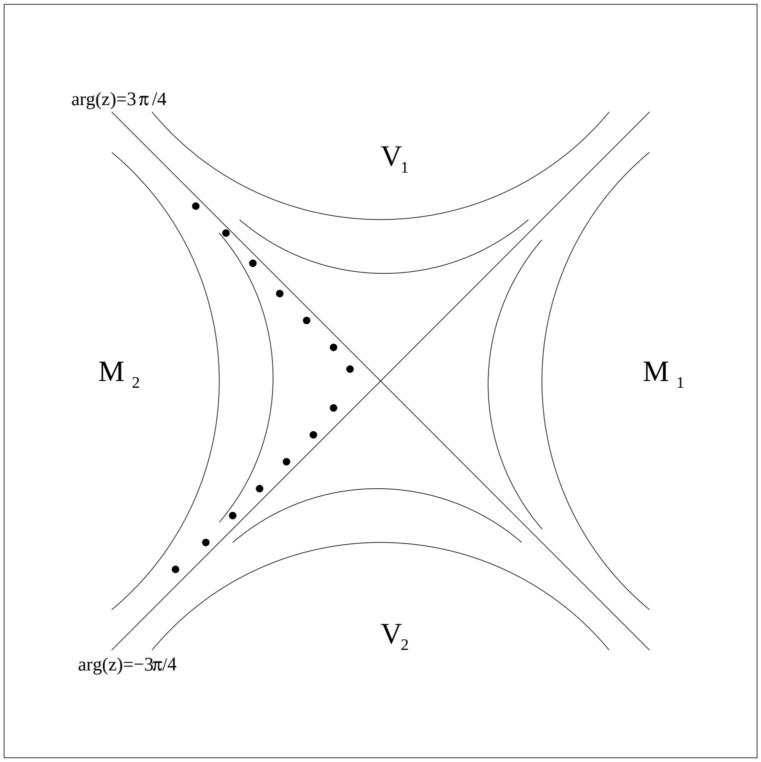

Figure 1: Curves lines of the landscape associated to $\varepsilon u' = z^2 - \varepsilon a - u^2$. the bullets indicate the poles close to the Stokes lines.

In the case of the equation

$$\varepsilon u' = z^2 - \varepsilon a - u^2 \tag{5}$$

it is well known that the overstability values are the odd positive integers. Hence our purpose in this article is to prove the conjecture of J.L Callot for equation (5).

2 Setting and notations

Our main tool is a transasymptotic analysis based on the recent work of O. Costin and R. Costin developed in [9] and [7]. By the elementary change of variables,

$$x = -z^2/\varepsilon \text{ and } y(x) = (2z)^{-1}u(z) + \frac{1}{2} - \frac{1-a}{4x},$$

equation (5) can be brought to the normalized form ([cost1], [cost2])

$$y' = -y - \frac{a}{2x}y + y^2 + \frac{3 - 4a + a^2}{16x^2} \tag{6}$$

2.1 Formal solution and summability

Theorem 1 *a) Equation (6) admits a unique formal solution* $\tilde{y}_0 \in x^{-1}\mathbb{C}[[x^{-1}]]$.

b) The formal solution $\tilde{y}_0$ *is Borel summable.*

c) Given an open sector of the complex x*-plane, of opening less than* π*, there exists an analytic solution of (6) which is asymptotic to* $\tilde{y}_0$ *in that sector. This solution is not unique.*

2.2 Transseries solutions

Proposition 2 *a) There exists a unique transseries which is a formal solution of (6) in the form*

$$\sum_{k\geq 0} e^{-kx}x^{-k\frac{a}{2}}\,\tilde{s}_k(x) \;\in\; \mathbb{C}\left[\left[e^{-x}x^{-\frac{a}{2}}, x^{-1}\right]\right] \tag{7}$$

where the $\tilde{s}_k$ *are formal series*

$$\tilde{s}_k \;=\; \sum_{r=0}^{\infty}\frac{\tilde{y}_{k,r}}{x^r} \in \mathbb{C}\left[\left[x^{-1}\right]\right] \;\; \textit{such that} \;\; \tilde{y}_{1,0} = 1.$$

Furthermore

$$\tilde{s}_0 = \;\tilde{y}_0.$$

b) For all $C \in \mathbb{C}$ *the formal series*

$$\tilde{y}(x, C) \;=\; \sum_{k\geq 0} C^k\, e^{-kx}x^{-k\frac{a}{2}}\,\tilde{s}_k(x) \;\in\; \mathbb{C}\left[\left[e^{-x}x^{-\frac{a}{2}}, x^{-1}\right]\right] \tag{8}$$

is a formal solution of (6).

c) For each formal solution of (6) of the form

$$\sum_{k\geq 0} e^{-kx}x^{-k\frac{a}{2}}\,\tilde{z}_k(x) \;\in\; \mathbb{C}\left[\left[e^{-x}x^{-\frac{a}{2}}, x^{-1}\right]\right]$$

there exists $C \in \mathbb{C}$ *such that*

$$\forall k \in \mathbb{N} \quad z_k = C^k\tilde{s}_k$$

Note 3 *A formal solution (8) is called transseries solution of (6).*

In [7], O. Costin introduced a generalized Borel sum of transseries denoted by $\mathcal{LB}$ which extends the classical Borel sum. He established a one-to-one correspondence between analytic solutions and generalized Borel sums of transseries. In [9] the authors study the position and the type of singularities formed by solutions when an irregular singular point (the infinity) of the equation is approached along an antistokes direction. The analysis of the singularities yields two-scale asymptotic expansions of solutions. The expansions have the form $y \sim \tilde{F}(x, \xi(x)) = \sum_{j=0}^{\infty} x^{-j} F_j(\xi(x))$, where $\xi(x) = Ce^{-x}x^{-\frac{a}{2}}$ and the functions F_m satisfy a recursive system of differential equations whose solutions are expressible by quadratures.
We can summarize the results proved in [9] in the following.

- The expansion above is valid in a region which includes the directions along which $y \longrightarrow 0$ and extends into regions where y is singular, as near as $O(e^{-const|x|})$ of these singularities.

- The singularities of y are grouped in regular arrays and are related to the singularities of F_0 in the following sense :
If ξ_s is an isolated singularity of F_0 and if $C \neq 0$ then $y(x)$ is singular at a distance at most $o(1)$ of $x_n \in \xi^{-1}(\xi_s)$ and

$$x_n \;=\; 2n\pi i - \frac{a}{2}\log(2n\pi i) + \log(C) - \log(\xi_s) + o(1) \qquad (9)$$

Remark 4 *Note that in the case of equation (6),* $F_0(\xi) = \dfrac{\xi}{\xi+1}$*, which has a unique pole at* $\xi = -1$.

3 Main results and proofs

3.1 The poles of a distinguished solution

We denote by $u_+(z) := -z$, one of the two slow curves. We consider the mountain containing the positive real axis and denote by u^* the distinguished solution associated to it. It is well known that, for each value of $a \not\simeq 2n+1$, $n \in \mathbb{N}$, the solution u^* has two lines of poles which occur infinitely close to the lines $\arg(z) = +\dfrac{3\pi}{4}$ and $\arg(z) = -\dfrac{3\pi}{4}$.
These two lines correspond in the new variable x to the lines $\arg(x) = \dfrac{5\pi}{2}$ and $\arg(x) = -\dfrac{\pi}{2}$.
Consequently, if y^* denotes the transseries solution of (6) corresponding to u^*, then

y^* may have poles infinitely close to the lines $\arg(x) = \frac{5\pi}{2}$ et $\arg(x) = -\frac{\pi}{2}$. This means that the transseries solution y^* is such that $C = 0$ in the first quadrant, and the solution is analytic in the sector $\{x : -\frac{\pi}{2} < arg(x) < \frac{5\pi}{2}\}$ of opening 3π.
In vertue of Theorem 4 ([7]), when $\arg(x)$ varies in this sector, the transseries solution y^* changes only through the constant C, and that change occurs when the Stokes line ($\mathbb{R}^+$) is crossed. Thus for y^* the constant C is given by

$$C(\phi) = \begin{cases} -\frac{1}{2}\mathbf{S} \quad for \quad \phi = 0 \\ 0 \quad for \quad 0 < \phi < 2\pi \\ \frac{1}{2}\mathbf{S} \quad for \quad \phi = 2\pi \\ +\mathbf{S} \quad for \quad 2\pi < \phi < \frac{5\pi}{2} \\ -\mathbf{S} \quad for \quad -\frac{\pi}{2} < \phi < 0 \end{cases} \tag{10}$$

where $\mathbf{S}$ is the Stokes constant of (6).
The poles of y^* near the antistokes line $i\mathbb{R}^+$ are then given by

$$x_n = 2n\pi i - \frac{a}{2}\log(2n\pi i) + \log(\mathbf{S}) - \log(-1) + o(1) \tag{11}$$

where $o(1)$ is a function which tends to 0 as n tends to ∞.

Definition 5 *Let V be a small enough sectorial neighberhood of $+i\infty$. The « Indicatrice des pôles » is the function*

$$\begin{aligned} I: \ \mathbb{C} &\longrightarrow \mathcal{P}(V) \\ a &\longmapsto I(a) = \{I_{N_0}(a)\,,\, I_{N_0+1}(a)\,,\,\ldots\} \end{aligned} \tag{12}$$

which, to each value of a, associates the poles $I_n(a) := x_n$ which are in V.

Theorem 6 *The odd positive integers are logarithmic singularities of the function « Indicatrice des pôles ».*

Remark 7 *1. The multiple values of $I(a)$ can be seen as the different values of one function obtained by analytic continuation.*

2. As the parameter a surrounds an odd positive integer, some given pole, say $x_n(a)$, moves to another pole : $x_n(ae^{2i\pi}) = x_{n+1}(a)$.

3.2 Proofs

3.2.1 Proof of theorem 6

It is clear that the dependance of the poles on a is tightly linked with the dependance of the possible poles and zeroes of $\mathbf{S}$ on a. An explicit calculation of $\mathbf{S}$, or a study of $\mathbf{S}$ as a function of a is then necessary to prove theorem 6.

Proposition 8 *The constante* $\mathbf{S}$ *is given by*

$$\mathbf{S}(\mathbf{a}) = \frac{(-1)^{\frac{1}{2}-a} 2^{\frac{1-a}{2}} \sqrt{\pi}}{\Gamma(\frac{1-a}{2})} \tag{13}$$

Theorem 6 is a consequence of the proposition above. Using (11) taking into account the proposition, the branches of the multivalued function « Indicatrice des pôles »are given by

$$I_n(a) = \; 2n\pi i - \frac{a}{2}\log(2n\pi i) + \log(\mathbf{S}) - \log(-1) + o(1).$$

The dominant term in $I_n(a)$ is an analytic function of a which admits the zeroes of $\mathbf{S}$ as logarithmic branch points. The zeroes of $\mathbf{S}$ are the simple poles of $\Gamma(\frac{1-a}{2})$ which are the odd positive integers. □

Proof of proposition 8

By the change of variables

$$\begin{cases} x = -t^2 \\ \\ y(x) = \dfrac{v'}{2tv} + \dfrac{1}{2t} + \dfrac{1-a}{4t^2} \end{cases} \tag{14}$$

equation (6) can be brought to

$$v'' = (t^2 - a)v \tag{15}$$

It is proven in [11] that equation (15) has a unique solution $v_0 = v_0(t, a)$ such that

- v_0 is an entire function of (t, a)
- v_0 admits an asymptotic representation

$$v_0(t, a) \sim t^{\frac{a-1}{2}} e^{\frac{-t^2}{2}} \left[1 + O(t^{-\frac{1}{2}})\right]$$

  uniformly on each compact set in the a-space as x tends to infinity in any closed subsector of the open sector

$$\Sigma_0 = \{t \in \mathbb{C}; |\arg(t)| < \frac{3\pi}{4}\}$$

- The functions v_k given by

$$v_k(t,a) = v_0(e^{-ik\frac{\pi}{2}}t, e^{-ik\pi}a)$$

are solutions of (15) for each $k \in \{0,1,2,3\}$.
Furthermore for each $k \in \{0,1,2,3\}$, the functions v_k and v_{k+1} are linearly independent (compare their asymptotics as $t \to \infty$, $\arg(t) = ki\pi$). Therefore, v_k is a linear combination of v_{k+1} and v_{k+2}.

- For $k = 0$ we have

$$v_0 = C_0(a)v_1 + \tilde{C}_0(a)v_2 \tag{16}$$

with

$$\begin{cases} \tilde{C}_0(a) = -e^{-\frac{i\pi}{2}(1+a)} \\ \\ C_0(a) = \frac{2^{\frac{1-a}{2}} e^{i\pi(\frac{a+1}{4})}\sqrt{\pi}}{\Gamma(\frac{1-a}{2})} \end{cases} \tag{17}$$

The calculation of the constant **S** is done by comparing two expressions of the variation of y^*. First, using the results above, one obtain an asymptotic representation of the variation of y^*, precisely

$$\operatorname{var} y^* \sim -\frac{C_0(a)}{\tilde{C}_0(a)}\frac{v_1}{v_2}.$$

On other hand, the variation of y^* is linked to the Stokes constant by the following statement :

Lemma 9 *The variation of y^* verifies*

$$\operatorname{var}(y^*(x)) \sim \mathbf{S}(a)e^{-x}x^{-a/2}$$

We have

$$y^*(x) = \frac{1}{2t}\frac{v_0'(t)}{v_0(t)} + \frac{1}{2} + \frac{1-a}{4t^2} \quad \text{where} \quad x = -t^2$$

hence

$$y^*(xe^{2i\pi}) = \frac{1}{2(-t)}\frac{v_0'(-t)}{v_0(-t)} + \frac{1}{2} + \frac{1-a}{4t^2} \quad \text{since} \quad xe^{2i\pi} = -(-t)^2$$

But as

$$v_0(-t) = v_2(t)$$

we get

$$y^*(xe^{2i\pi}) = \frac{1}{2t}\frac{v_2'(t)}{v_2(t)} + \frac{1}{2} + \frac{1-a}{4t^2}.$$

Using (16) one obtains

$$\mathrm{var}\, y^*(x) = \frac{1}{2t}\frac{C(a)}{\tilde{C}(a)}\frac{v_1}{v_2}\left[\frac{\dfrac{v_2'}{v_2} - \dfrac{v_1'}{v_1}}{1 + \dfrac{C(a)}{\tilde{C}(a)}\dfrac{v_1}{v_2}}\right].$$

Since

$$\frac{v_2(t)'}{v_2(t)} = \frac{a-1}{2t} - t + O(t^{-3/2})$$
$$\frac{v_1(t)'}{v_1}t - \frac{a+1}{2t} + O(t^{-3/2})$$

and

$$\frac{v_1(t)}{v_2(t)} \sim (-1)^{\frac{1-a}{2}} e^{i\pi\frac{a+1}{4}} t^{-a} e^{t^2} = (-1)^{\frac{1}{2}-a} e^{i\pi\frac{a+1}{4}} x^{-\frac{a}{2}} e^{-x}$$

we get

$$\mathrm{var}\, y^*(x) \sim -\frac{C(a)}{\tilde{C}(a)}\frac{v_1}{v_2}.$$

In vertue of (17),

$$-\frac{C(a)}{\tilde{C}(a)} = \frac{2^{\frac{1-a}{2}} e^{-i\pi(\frac{a+1}{4})}\sqrt{\pi}}{\Gamma(\frac{1-a}{2})}$$

hence

$$\mathrm{var}\, y^*(x) \sim \frac{(-1)^{\frac{1}{2}-a} 2^{\frac{1-a}{2}}\sqrt{\pi}}{\Gamma(\frac{1-a}{2})} x^{-\frac{a}{2}} e^{-x}. \tag{18}$$

Proof of lemma 9

Let x be such that $0 < \arg(x) < \pi/2$. Then in vertue of (10)

$$y^*(x) = \mathcal{LB}(\tilde{y}_0(x))$$

and

$$y^*(xe^{2i\pi}) = \sum_{k=0}^{k=\infty} \mathbf{S}^k(a) e^{-kx} x^{-ka/2} \mathcal{LB}(\tilde{y}_k(x)).$$

Because

$$\mathcal{LB}(\tilde{y}_k(x)) \sim \tilde{y}_k(x) \ \ \forall k \ \text{ and } \ \tilde{y}_{1,0} = 1$$

we obtain

$$y^*(xe^{2i\pi}) - y^*(x) \sim \mathbf{S}(a)e^{-x}x^{-a/2} \tag{19}$$

□

Comparing (18) and (19) one obtains

$$\mathbf{S(a)} = \frac{(-1)^{\frac{1}{2}-a}2^{\frac{1-a}{2}}\sqrt{\pi}}{\Gamma(\frac{1-a}{2})}.$$

□

3.2.2 Sketch of proof of theorem 1

For *a)* we first rewrite equation (6) in the form :

$$y = (1+a/2x)^{-1}\left[-y' + y^2 + \frac{3-4a+a^2}{16x^2}\right]$$

We denote by $E := x^{-1}\mathbb{C}[[x^{-1}]]$ and define an operator $\mathcal{F}$

$$\begin{aligned} \mathcal{F} \;:\; & E \longrightarrow E \\ & y \longmapsto \mathcal{F}(y) \end{aligned}$$

such that

$$\mathcal{F}(y)(x) = (1+a/2x)^{-1}\left[-y' + y^2 + \frac{3-4a+a^2}{16x^2}\right].$$

We endow E by the distance $d : (y_1, y_2) \longmapsto 0$ if $y_1 = y_2$ and $e^{-val(y_1-y_2)}$ if not. We prove that (E, D) is complete and $\mathcal{F}$ is a contraction.
For *b)* a complete proof in a more general setting is given in [5] and also in [2].
Item *c)* is a result of classical asymptotic and follows from the proof of Theorem 12.1 of [12].

3.2.3 Sketch of proof of proposition 2

By a formal substitution of $y = \sum_{k=0}^{k=\infty} y_k e^{-kx}$ in equation (6) we obtain

$$\begin{cases} y_0' = \dfrac{3-4a+a^2}{16x^2} - y_0 - \dfrac{a}{2x}y_0 + y_0^2 \\ y_1' = -\dfrac{a}{2x}y_1 + 2y_0y_1 \\ y_k' = (k-1)y_k - \dfrac{a}{2x}y_k + 2y_0^{l-1}y_k + \mathbf{P}_k(y_0, y_1, \ldots y_{k-1}) \text{ for } k \geq 2 \end{cases}$$

where $\mathbf{P}_k(y_0, y_1, \dots y_{k-1})$ is polynomial on $y_1, y_2, \dots y_{k-1}$ whose coefficients are in $\mathbb{C}[[x^{-1}, y_0]]$ and satisfy

$$\mathbf{P}_k(y_0, Cy_1, C^2 y_2, \dots C^{k-1} y_{k-1}) = C^k \mathbf{P}_k(y_0, y_1, \dots y_{k-1}) \tag{20}$$

In order to find the equations satisfied by $\tilde{s}_k$, we replace in the system above y_k by $x^{-ka/2}\tilde{s}_k$ and we obtain exactly the same system. Hence the statement *a)* can be proved by studying the equations for each k. *b)* is a consequence of (20) and *c)* results from *a)* and *b)*.

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