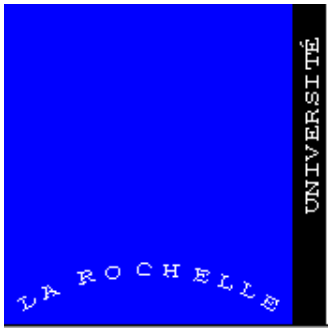

Prépublications du Département de Mathématiques

Université de La Rochelle
Avenue Michel Crépeau
17042 La Rochelle Cedex 1
http://www.univ-lr.fr/labo/lmca/

Existence results to elliptic systems with nonstandard growth conditions

Abdallah El Hamidi

Mai 2004

Classification:
Mots clés: Nonlinear elliptic systems, nonstandard growth, generalized Lebesgue-Sobolev spaces, Mountain pass geometry.

2004/12

Existence Results to Elliptic Systems with Nonstandard Growth Conditions

A. El Hamidi

Université de La Rochelle
Laboratoire de Mathématiques et Applications
Avenue Michel Crépeau
17042 La Rochelle
France

Abstract

Nonstandard growth conditions in partial differential equations have been the subject of recent developments in elastic mechanics and electrorheological fluid dynamics [15,16,19]. In this work, elliptic systems with nonstandard growth conditions are studied. Existence and multiplicity results, under growth conditions on the reaction terms, are established.

Key words: Nonlinear elliptic systems, nonstandard growth conditions, generalized Lebesgue-Sobolev spaces, mountain pass geometry

1 Introduction

Differential and partial differential equations with nonstandard growth conditions have received specific attention in recent decades. The interest played by such growth conditions in elastic mechanics and electrorheological fluid dynamics has been highlighted in many physical and mathematical works. We can refer the reader to M. Ruzicka [15,16], V. Zhikov [19] and the refernces therein.
The present paper deals with existence results to elliptic systems of gradient type with

Email address: aelhamid@univ-lr.fr (A. El Hamidi).

nonstandard growth conditions. More precisely, we consider the following system

$$\begin{cases} -\text{div}\left(|\nabla u|^{p(x)-2}\nabla u\right) = \frac{\partial F}{\partial u}(x,u,v) & \text{in } \Omega \\ -\text{div}\left(|\nabla v|^{q(x)-2}\nabla v\right) = \frac{\partial F}{\partial v}(x,u,v) & \text{in } \Omega \\ u = 0,\ v = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with a smooth boundary $\partial\Omega$, $N \geq 2$, $(p,q) \in C(\overline{\Omega})^2$, $p(x) > 1$, $q(x) > 1$, for every $x \in \Omega$. The function F is assumed to be continuous in $x \in \overline{\Omega}$ and of class C^1 in u, $v \in \mathbb{R}$.
Elliptic systems with standard growth conditions have been the subject of a sizeable literature. We refer to the excellent survey article of D. G. de Figueiredo [2].
Here, problem (1) is stated in the framework of the generalized Sobolev space $W_0^{1,p(x)}(\Omega) \times W_0^{1,q(x)}(\Omega)$ which will be briefly described in the following section.

2 Preliminary results and notations

For the reader's convenience, we recall some background facts concerning the generalized Lebesgue-Sobolev spaces and introduce some notations used below. We can refer the reader to the book of J. Musielak [14] and the papers of X. L. Fan & *al.* [4,5,6,7,8,9,10], P. Marcellini [11,12] and D. Edmunds & J. Rákosník [3].

Let

$$C_+(\overline{\Omega}) = \{h \in C(\overline{\Omega}) :\ h(x) > 1 \text{ for every } x \in \overline{\Omega}\},$$

$$h^+ = \max\{h(x),\ x \in \overline{\Omega}\},\quad h^- = \min\{h(x),\ x \in \overline{\Omega}\},\quad \text{for every } h \in C_+(\overline{\Omega}),$$

and introduce, for $p \in C_+(\overline{\Omega})$, the space

$$L^{p(x)}(\Omega) = \left\{u \text{ measurable real-valued function in } \Omega : \int_\Omega |u(x)|^{p(x)}dx < +\infty\right\}.$$

Endowed with the norm

$$|u|_{L^{p(x)}(\Omega)} = \inf\left\{\lambda > 0 :\ \int_\Omega \left|\frac{u(x)}{\lambda}\right|^{p(x)} dx \leq 1\right\},$$

the space $L^{p(x)}(\Omega)$ possesses the Banach structure. The dual space of $L^{p(x)}(\Omega)$ is $L^{p'(x)}(\Omega)$, where $1/p(x) + 1/p'(x) = 1$, $\forall x \in \Omega$.

As in the standard case, the space $W^{1,p(x)}(\Omega)$ is defined as follows

$$W^{1,p(x)}(\Omega) = \left\{u \in L^{p(x)} :\ |\nabla u| \in L^{p(x)}\right\}.$$

Endowed with its natural norm

$$||u||_{W^{1,p(x)}(\Omega)} := |u|_{L^{p(x)}(\Omega)} + |\nabla u|_{L^{p(x)}(\Omega)}$$

$W^{1,p(x)}(\Omega)$ is a separable and reflexive Banach space. If we denote by $W_0^{1,p(x)}(\Omega)$ the closure of $C_0^\infty(\Omega)$ in $W^{1,p(x)}(\Omega)$, then the Poincaré inequality

$$|u|_{L^{p(x)}(\Omega)} \leq C|\nabla u|_{L^{p(x)}(\Omega)}$$

holds true. Moreover, $W_0^{1,p(x)}(\Omega)$, endowed with the norm $|\nabla u|_{L^{p(x)}(\Omega)}$, is a separable and reflexive Banach space. Notice that the inequality

$$\int_\Omega |u|^{p(x)} dx \leq C \int_\Omega |\nabla u|^{p(x)} dx$$

is false in general [9]. Let us define, for every $x \in \Omega$,

$$p^*(x) = \begin{cases} \frac{Np(x)}{N-p(x)} \text{ if } p(x) < N, \\ +\infty \quad \text{if } p(x) \geq N. \end{cases}$$

As in the classical case, the embedding $W_0^{1,p(x)}(\Omega) \subset L^{r(x)}(\Omega)$ is continuous (resp. compact) if $r(x) \leq p^*(x)$ (resp. $r(x) < p^*(x)$), for all $x \in \overline{\Omega}$. For the other properties satisfied by the the generalized Lebesgue and Sobolev spaces, we refer the reader to [9]. However, we recall the following

Lemma 1 (see [9]). *If we denote*

$$\rho(u) = \int_\Omega |u|^{p(x)} \, dx, \ \ \forall \, u \in L^{p(x)}(\Omega)$$

then

(i) $|u|_{L^{p(x)}(\Omega)} < 1 \ (resp. \ = 1; > 1) \Longleftrightarrow \rho(u) < 1 \ (resp. \ = 1; > 1)$;

(ii) $|u|_{L^{p(x)}(\Omega)} > 1 \Longrightarrow |u|^{p^-}_{L^{p(x)}(\Omega)} \leq \rho(u) \leq |u|^{p^+}_{L^{p(x)}(\Omega)}$;

(iii) $|u|_{L^{p(x)}(\Omega)} < 1 \Longrightarrow |u|^{p^+}_{L^{p(x)}(\Omega)} \leq \rho(u) \leq |u|^{p^-}_{L^{p(x)}(\Omega)}$;

(iv) $|u|_{L^{p(x)}(\Omega)} \longrightarrow 0 \ (resp. \to +\infty) \Longleftrightarrow \rho(u) \longrightarrow 0 \ (resp. \longrightarrow +\infty)$;

(v) $\rho\left(u/|u|_{L^{p(x)}(\Omega)}\right) = 1$.

For every (u, v) and (φ, ψ) in $W := W_0^{1,p(x)}(\Omega) \times W_0^{1,q(x)}(\Omega)$, let

$$\mathcal{F}(u, v) := \int_\Omega F(x, u, v) \, dx.$$

Then,

$$\mathcal{F}'(u, v)(\varphi, \psi) = D_1\mathcal{F}(u, v)(\varphi) + D_2\mathcal{F}(u, v)(\psi),$$

where

$$D_1\mathcal{F}(u, v)(\varphi) = \int_\Omega \frac{\partial F}{\partial u}(x, u, v)\varphi \, dx \quad \text{and} \quad D_2\mathcal{F}(u, v)(\psi) = \int_\Omega \frac{\partial F}{\partial v}(x, u, v)\psi \, dx.$$

The Euler-Lagrange functional associated to (1) is given by

$$J(u, v) := \int_\Omega \frac{1}{p(x)}|\nabla u|^{p(x)} \, dx + \int_\Omega \frac{1}{q(x)}|\nabla v|^{q(x)} \, dx - \mathcal{F}(u, v).$$

It is easy to verify that $J \in C^1(W, \mathbb{R})$ and that

$$J'(u,v)(\varphi,\psi) = D_1 J(u,v)(\varphi) + D_2 J(u,v)(\psi), \tag{2}$$

where

$$D_1 J(u,v)(\varphi) = \int_\Omega |\nabla u|^{p(x)-2}\nabla u \nabla \varphi \, dx - D_1 \mathcal{F}(u,v)(\varphi),$$
$$D_2 J(u,v)(\psi) = \int_\Omega |\nabla v|^{q(x)-2}\nabla v \nabla \psi \, dx - D_2 \mathcal{F}(u,v)(\psi).$$

Let us choose on W the norm $||\,.\,||$ defined by

$$||(u,v)|| := \max\left\{||u||_p + ||v||_q\right\},$$

where $||\,.\,||_p$ (resp. $||\,.\,||_q$) is the norm of $W_0^{1,p(x)}(\Omega)$ (resp. $W_0^{1,q(x)}(\Omega)$). The dual space of W will be denoted by W^* and $||\,.\,||_*$ will stand for its norm. Therefore

$$||J'(u,v)||_* = ||D_1 J(u,v)||_{*,p} + ||D_2 J(u,v)||_{*,q},$$

where $W^{-1,p'(x)}(\Omega)$ (resp. $W^{-1,q'(x)}(\Omega)$) is the dual space of $W_0^{1,p(x)}(\Omega)$ (resp. $W_0^{1,q(x)}(\Omega)$) and $||\,.\,||_{*,p}$ (resp. $||\,.\,||_{*,q}$) is its norm. Finally, $|\,.\,|_r$ will stand for the norm on $L^{r(x)}(\Omega)$, for every $r \in C_+(\overline{\Omega})$.

Throughout this paper, the letters c, c_i, $i = 1, 2, \ldots$, denote positive constants which may vary from line to line but are independent of the terms which will take part in any limit process.

3 Existence results

Before stating our results, we introduce some natural growth hypotheses on the right hand side of (1). Theses hypotheses will insure the mountain pass geometry and the Palais-Smale condition for the Euler-Lagrange functional J. We limit ourselves to the subcritical case, *i.e.* we assume that

(H_1) $F(x,s,t) \le c_1 + c_2|s|^{p_1(x)} + c_3|t|^{q_1(x)} + c_4|s|^{\alpha(x)}|t|^{\beta(x)},\ \forall\,(x,s,t) \in \Omega \times \mathbb{R}^2$, where $(p_1, q_1, \alpha, \beta) \in C_+(\overline{\Omega})^4$ and

$$p_1 < p^*,\ q_1 < q^*,\ \frac{\alpha}{p^*} + \frac{\beta}{q^*} < 1 \text{ in } \overline{\Omega},\ p_1^-, \alpha^- > p^+ \text{ and } q_1^-, \beta^- > q^+.$$

Precise that under the hypothesis (H_1), the operator $\mathcal{F}' : W \longrightarrow W^*$ is compact [9]. To guarantee the Palais-Smale condition for the functional J, we will assume moreover the following

(H_2) $\exists\, M > 0,\ \exists\, \theta_1 > p^+, \exists\, \theta_2 > q^+ :\ \forall\, x \in \Omega,\ \forall\,(s,t) \in \mathbb{R}^2 :\ |s|^{\theta_1} + |t|^{\theta_2} \ge 2M$, on has

$$0 < F(x,s,t) \le \frac{s}{\theta_1}\frac{\partial F}{\partial s}(x,s,t) + \frac{t}{\theta_2}\frac{\partial F}{\partial t}(x,s,t).$$

Finally, we assume

(H_3) $F(x,s,t) = o\left(|s|^{p^+} + |t|^{q^+}\right)$ as $(s,t) \longrightarrow (0,0)$, uniformly w.r.t $x \in \Omega$.

As we will show later, the hypotheses (H_1) and (H_3) imply the mountain pass geometry for the functional J.

Lemma 2 *Let (u_n, v_n) be a Palais-Smale sequence for the Euler-Lagrange functional J. If (H_2) is satisfied then (u_n, v_n) is bounded.*

PROOF. Let (u_n, v_n) be a Palais-Smale sequence for the functional J. This means that $J(u_n, v_n)$ is bounded and $||J'(u_n, v_n)||_* \longrightarrow 0$ as n goes to $+\infty$. Then, there is a positive constant c such that

$$\begin{aligned}
c &\geq J(u_n, v_n) \\
&= \int_\Omega \frac{1}{p(x)} |\nabla u_n|^{p(x)} dx + \int_\Omega \frac{1}{q(x)} |\nabla v_n|^{q(x)} dx - \int_\Omega F(x, u_n, v_n)\, dx \\
&\geq \int_\Omega \left(\frac{1}{p(x)} |\nabla u_n|^{p(x)} - \frac{u_n}{\theta_1} \frac{\partial F}{\partial u}(x, u_n, v_n) \right) dx + \\
&\quad \int_\Omega \left(\frac{1}{q(x)} |\nabla v_n|^{q(x)} - \frac{v_n}{\theta_2} \frac{\partial F}{\partial v}(x, u_n, v_n) \right) dx - c_1
\end{aligned}$$

where c_1 is some positive constant. Then,

$$\begin{aligned}
c &\geq \left(\frac{1}{p^+} - \frac{1}{\theta_1} \right) \int_\Omega |\nabla u_n|^{p(x)} dx + \left(\frac{1}{q^+} - \frac{1}{\theta_2} \right) \int_\Omega |\nabla v_n|^{q(x)} dx + \\
&\quad \frac{1}{\theta_1} \int_\Omega \left(|\nabla u_n|^{p(x)} - u_n \frac{\partial F}{\partial u}(x, u_n, v_n) \right) dx + \\
&\quad \frac{1}{\theta_2} \int_\Omega \left(|\nabla v_n|^{q(x)} - v_n \frac{\partial F}{\partial v}(x, u_n, v_n) \right) dx - c_2 \\
&\geq \left(\frac{1}{p^+} - \frac{1}{\theta_1} \right) \int_\Omega |\nabla u_n|^{p(x)} dx + \left(\frac{1}{q^+} - \frac{1}{\theta_2} \right) \int_\Omega |\nabla v_n|^{q(x)} dx - \\
&\quad \frac{1}{\theta_1} ||D_1 J(u_n, v_n)||_{*,p} ||u_n||_p - \frac{1}{\theta_2} ||D_2 J(u_n, v_n)||_{*,q} ||v_n||_q - c_2.
\end{aligned}$$

Now, suppose that the sequence (u_n, v_n) is not bounded. Without loss of generality, we may assume $||u_n||_p \geq ||v_n||_q$.

Therefore, for n large enough, we get

$$c \geq \left(\frac{1}{p^+} - \frac{1}{\theta_1} \right) ||u_n||_p^{p^-} - \left(\frac{1}{\theta_1} ||D_1 J(u_n, v_n)||_{*,p} - \frac{1}{\theta_2} ||D_2 J(u_n, v_n)||_{*,q} \right) ||u_n||_p.$$

But, this can not hold true since $p^- > 1$. Hence, the sequence (u_n, v_n) is bounded. □

In the following lemma, we show a compactness result. More precisely, we show that every bounded Plais-Smale sequence for the functional J contains a Cauchy subsequence. In the simple case where the functions p and q are constant, the extraction of

Cauchy subsequences, based on the inequalities (3), is very standard. Here, we adapt this method to the general case where p and q are in $C_+(\overline{\Omega})$.

Lemma 3 *Let (u_n, v_n) be a bounded Palais-Smale sequence for the Euler-Lagrange functional J. If (H_1) is satisfied then (u_n, v_n) contains a convergent subsequence.*

PROOF. Let (u_n, v_n) be a bounded Palais-Smale sequence for the functional J. Then there is a subsequence still denoted by (u_n, v_n) which converges weakly in W. We recall the well known inequalities

$$\begin{cases} |x-y|^\gamma \leq 2^\gamma \left(|x|^{\gamma-2}x - |y|^{\gamma-2}y\right)\cdot(x-y) & \text{if } \gamma \geq 2 \\ |x-y|^2 \leq \left(|x|+|y|\right)^{2-\gamma}\left(|x|^{\gamma-2}x - |y|^{\gamma-2}y\right)\cdot(x-y) & \text{if } 1<\gamma<2 \end{cases} \tag{3}$$

for every x and y in $\mathbb{R}^N$, where $\cdot$ denotes the standard inner product in $\mathbb{R}^N$. We show that (u_n, v_n) contains a Cauchy subsequence. Let us define

$$\begin{aligned} \mathcal{U}_p &= \{x\in\Omega \,:\, p(x)\geq 2\} \text{ and } \mathcal{V}_p = \{x\in\Omega \,:\, 1<p(x)<2\}, \\ \mathcal{U}_q &= \{x\in\Omega \,:\, q(x)\geq 2\} \text{ and } \mathcal{V}_q = \{x\in\Omega \,:\, 1<q(x)<2\}. \end{aligned}$$

For every $x\in\Omega$, we set

$$\begin{aligned} \Phi_{n,k} &= \left(|\nabla u_n|^{p(x)-2}\nabla u_n - |\nabla u_k|^{p(x)-2}\nabla u_k\right)\cdot(\nabla u_n - \nabla u_k), \\ \Psi_{n,k} &= \left(|\nabla u_n| + |\nabla u_k|\right)^{2-p(x)}. \end{aligned}$$

Then,

$$\begin{aligned} \int_{\mathcal{U}_p} |\nabla u_n - \nabla u_k|^{p(x)}dx &\leq c\int_\Omega \Phi_{n,k}\,dx, \\ \int_{\mathcal{V}_p} |\nabla u_n - \nabla u_k|^{p(x)}dx &\leq \int_\Omega \Phi_{n,k}^{\frac{p(x)}{2}}\Psi_{n,k}^{(2-p(x))\frac{p(x)}{2}}dx. \end{aligned}$$

Recall that

$$\begin{aligned} \int_\Omega \Phi_{n,k}\,dx &= \left(D_1J(u_n,v_n) - D_1J(u_k,v_k) + D_1\mathcal{F}(u_n,v_n) - D_1\mathcal{F}(u_k,v_k)\right)(u_n-u_k) \\ &\leq \left(||D_1J(u_n,v_n)||_{*,p} + ||D_1J(u_k,v_k)||_{*,p}\right)||u_n-u_k||_p + \\ &\quad \left(||D_1\mathcal{F}(u_n,v_n) - D_1\mathcal{F}(u_k,v_k)||_{*,p}\right)||u_n-u_k||_p. \end{aligned}$$

Since $||J'(u_m,v_m)||_* \longrightarrow 0$, as m goes to $+\infty$, and $\mathcal{F}'$ is compact then we can consider that

$$0 \leq \int_\Omega \Phi_{n,k}\,dx < 1.$$

If

$$\int_\Omega \Phi_{n,k}\,dx = 0,$$

using the fact that $\Phi_{n,k}\geq 0$ in Ω, we obtain that $\Phi_{n,k}=0$.

If

$$0 < \int_\Omega \Phi_{n,k}\,dx < 1. \tag{4}$$

Thanks to the Young inequality and (4), we conclude

$$\int_{\mathcal{V}_p} \Phi_{n,k}^{\frac{p(x)}{2}} \left(\int_{\mathcal{V}_p} \Phi_{n,k}(y)\, dy \right)^{\frac{-1}{2}} \Psi_{n,k}^{(2-p(x))\frac{p(x)}{2}} dx \leq \int_{\mathcal{V}_p} \left(\Phi_{n,k} \left(\int_{\mathcal{V}_p} \Phi_{n,k}(y)\, dy \right)^{\frac{-1}{p(x)}} + \Psi_{n,k}^{p(x)} \right) dx$$
$$\leq 1 + \int_\Omega \Psi_{n,k}^{p(x)}\, dx.$$

Hence,

$$\int_{\mathcal{V}_p} |\nabla u_n - \nabla u_k|^{p(x)} dx \leq \left(\int_\Omega \Phi_{n,k}\, dx \right)^{\frac{1}{2}} \left(1 + \int_\Omega \Psi_{n,k}^{p(x)}\, dx \right).$$

Notice that $\int_\Omega \Psi_{n,k}^{p(x)}\, dx$ is bounded uniformly w.r.t n and k. Using again the fact that $||J'(u_m, v_m)||_* \longrightarrow 0$, as m tends to $+\infty$, and $\mathcal{F}'$ is compact, we obtain, up to a subsequence,

$$\lim_{n,k\to+\infty} \int_\Omega \Phi_{n,k}\, dx = 0.$$

Therefore, passing if necessary to a subsequence, we have

$$\lim_{n,k\to+\infty} \int_\Omega |\nabla u_n - \nabla u_k|^{p(x)} dx = 0.$$

Applying the same arguments, we find a subsequence of (u_n, v_n) such that

$$\lim_{n,k\to+\infty} \int_\Omega |\nabla v_n - \nabla v_k|^{q(x)} dx = 0.$$

Hence, up to a subsequence, one has

$$\lim_{n,k\to+\infty} ||(u_n, v_n) - (u_k, v_k)|| = 0,$$

which implies that (u_n, v_n) contains a Cauchy subsequence and therefore contains a strongly convergent subsequence. This ends the proof.

Remark 4 *The proof of the previous lemma can be showed also via the Lebesgue dominated convergence theorem.*

Now, we can state the following existence result

Theorem 5 *If the hypotheses* $(\mathrm{H}_1) - (\mathrm{H}_3)$ *hold true, then the problem (1) has at least one weak solution.*

PROOF. First, let λ, μ in $C_+(\overline{\Omega})$ satisfying $\lambda(x) < \mu^*(x)$ in Ω. Using the continuous embedding $W_0^{1,\mu(x)}(\Omega) \subset L^{\lambda(x)}(\Omega)$ and Lemma 1, we get: $\exists\ \delta_1, \delta_2 :\ 0 < \delta_1, \delta_2 < 1$ such that

$$\forall\, f \in W_0^{1,\mu(x)}(\Omega),\quad ||f||_{W_0^{1,\mu(x)}(\Omega)} \leq \delta_1 \Longrightarrow ||f||_{L^{\lambda(x)}(\Omega)} \leq \delta_2. \tag{5}$$

On the other hand, let $\widetilde{\alpha}$, $\widetilde{\beta}$ be two continuous and positive functions on $\overline{\Omega}$ such that

$$\frac{\alpha(x) + \widetilde{\alpha}(x)}{p^*(x)} + \frac{\beta(x) + \widetilde{\beta}(x)}{q^*(x)} = 1,\quad \forall\, x \in \overline{\Omega}.$$

For $||(u,v)|| << 1$, using the Young inequality, Lemma 1 and (5), we obtain

$$\begin{aligned}\int_\Omega |u|^{\alpha(x)}|v|^{\beta(x)}\,dx &\leq \left(\int_\Omega |u|^{\alpha(x)}\,dx\right)^{\left(\frac{p^*}{\alpha+\beta}\right)^-} + \left(\int_\Omega |v|^{\beta(x)}\,dx\right)^{\left(\frac{q^*}{\beta+\beta}\right)^-}\\ &\leq \int_\Omega |u|^{\alpha(x)}\,dx + \int_\Omega |v|^{\beta(x)}\,dx\\ &\leq c\left(||u||_p^{\alpha^-} + ||v||_q^{\beta^-}\right).\end{aligned}$$

Finally, notice that under the hypothese (H_1), we have the continuous embeddings $W_0^{1,p(x)}(\Omega) \subset L^{p^+}(\Omega)$ and $W_0^{1,q(x)}(\Omega) \subset L^{q^+}(\Omega)$. That is, $\exists\, C_1 > 0$, $\exists\, C_2 > 0$ such that

$$||u||_{L^{p^+}(\Omega)} \leq C_1||u||_{W_0^{1,p(x)}(\Omega)} \text{ and } ||v||_{L^{q^+}(\Omega)} \leq C_2||v||_{W_0^{1,q(x)}(\Omega)}.$$

Now, let $\varepsilon > 0$ such that

$$\varepsilon \max(C_1^{p^+}, C_2^{q^+}) \leq \frac{1}{2}\min(p^+, q^+).$$

It follows, from (H_1) and (H_3) that $\forall\,(x,s,t) \in \Omega \times \mathbb{R}^2$

$$F(x,s,t) \leq \varepsilon\left(|s|^{p^+} + |t|^{q^+}\right) + C(\varepsilon)\left(|s|^{p_1(x)} + |t|^{q_1(x)} + |s|^{\alpha(x)}|t|^{\beta(x)}\right).$$

Therefore, for $||(u,v)||$ sufficiently small, we get

$$\begin{aligned}J(u,v) \geq{}& \frac{1}{p^+}||u||_p^{p^+} - \varepsilon C_1^{p^+}||u||_p^{p^+} + \frac{1}{q^+}||v||_q^{q^+} - \varepsilon C_2^{q^+}||v||_q^{q^+} -\\ & C(\varepsilon)\int_\Omega \left(|u|^{p_1(x)} + |v|^{q_1(x)} + |u|^{\alpha(x)}|v|^{\beta(x)}\right)dx\\ \geq{}& \frac{||u||_p^{p^+}}{2p^+} + \frac{||v||_q^{q^+}}{2q^+} - C(\varepsilon)\left(||u||_p^{p_1^-} + ||v||_q^{q_1^-} + c||u||_p^{\alpha^-} + c||v||_q^{\beta^-}\right).\end{aligned}$$

Since $p_1^-, \alpha^- > p^+$ and $q_1^-, \beta^- > q^+$, there is $r > 0$ and $c' > 0$ such that $J(u,v) \geq c'$ for every $(u,v) \in W$ satisfying $||(u,v)|| = r$.

On the other hand, we claim that the assumption (H_2) implies the following assertion: for every $x \in \overline{\Omega}$, $s,t \in \mathbb{R}$, the inequality

$$F(x,s,t) \geq c\left(\left(|s|^{\theta_1} + |t|^{\theta_2}\right) - 1\right)$$

holds true. Indeed, consider the compact subset K of $\mathbb{R}^2$ defined by

$$K := \left\{(s,t) \in \mathbb{R}^2 \;:\; \frac{|s|^{\theta_1} + |t|^{\theta_2}}{2} = 1\right\}.$$

For every $(s,t) \in K$, we introduce the function

$$G(x,\tau) := F\left(x, |\tau|^{\frac{1}{\theta_1}}s, |\tau|^{\frac{1}{\theta_2}}t\right)$$

defined on $\overline{\Omega} \times \mathbb{R}$. Then,

$$\tau \frac{\partial G}{\partial \tau} = \frac{s}{\theta_1} |\tau|^{\frac{1}{\theta_1}} \partial_2 F \left(x, |\tau|^{\frac{1}{\theta_1}} s, |\tau|^{\frac{1}{\theta_2}} t \right) + \frac{t}{\theta_2} |\tau|^{\frac{1}{\theta_2}} \partial_3 F \left(x, |\tau|^{\frac{1}{\theta_1}} s, |\tau|^{\frac{1}{\theta_2}} t \right),$$

where ∂_i denotes the partial derivative of F w.r.t its i-th variable. Therefore, for $|\tau| \geq M$, we get

$$\tau \frac{\partial G}{\partial \tau} \geq G(x, \tau) > 0$$

which implies that $G(x, \tau) \geq \frac{G(x,M)}{M} |\tau|$. This means that

$$F \left(x, |\tau|^{\frac{1}{\theta_1}} s, |\tau|^{\frac{1}{\theta_2}} t \right) \geq \frac{F \left(x, M^{\frac{1}{\theta_1}} s, M^{\frac{1}{\theta_2}} t \right)}{M} |\tau|, \ \forall \, |\tau| \geq M.$$

Set

$$c_1 := \min_{x \in \overline{\Omega}} \min_{(s,t) \in K} \frac{F \left(x, M^{\frac{1}{\theta_1}} s, M^{\frac{1}{\theta_2}} t \right)}{M} > 0.$$

Then $F \left(x, |\tau|^{\frac{1}{\theta_1}} s, |\tau|^{\frac{1}{\theta_2}} t \right) \geq c_1 |\tau| - c_2, \ \forall \, \tau \in \mathbb{R}$. Moreover, every $(u, v) \in \mathbb{R}^2$ can be rewritten as

$$(u, v) = \left(\left(|u|^{\theta_1} + |v|^{\theta_2} \right)^{\frac{1}{\theta_1}} u', \left(|u|^{\theta_1} + |v|^{\theta_2} \right)^{\frac{1}{\theta_2}} v' \right),$$

where $(u', v') \in K$. Therefore, there is $c > 0$ such that $F(x, s', t') \geq c(|s'|^{\theta_1} + |t'|^{\theta_2} - 1), \ \forall (s', t') \in \mathbb{R}^2$. This achieves our claim.

Now, Let $\widetilde{u} \in W_0^{1,p(x)}(\Omega) \setminus \{0\}$, $\widetilde{v} \in W_0^{1,q(x)}(\Omega) \setminus \{0\}$ and $t > 1$. Then,

$$\begin{aligned} J(t\widetilde{u}, t\widetilde{v}) &= \int_\Omega \frac{t^{p(x)} |\nabla \widetilde{u}|^{p(x)}}{p(x)} \, dx + \int_\Omega \frac{t^{q(x)} |\nabla \widetilde{v}|^{q(x)}}{q(x)} \, dx - \int_\Omega F(x, t\widetilde{u}, t\widetilde{v}) \, dx \\ &\leq t^{p^+} \int_\Omega \frac{|\nabla \widetilde{u}|^{p(x)}}{p(x)} \, dx + t^{q^+} \int_\Omega \frac{|\nabla \widetilde{v}|^{q(x)}}{q(x)} \, dx - \\ &\quad c t^{\theta_1} \int_\Omega |\widetilde{u}|^{\theta_1} dx - c t^{\theta_2} \int_\Omega |\widetilde{v}|^{\theta_2} dx - c|\Omega|. \end{aligned}$$

Therefore,

$$\lim_{t \to +\infty} J(t\widetilde{u}, t\widetilde{v}) = -\infty$$

which implies that J possesses the mountain pass geometry. Applying Lemma 2 and Lemma 3, we conclude that J has at least one nontrivial critical point [13,17,18]. This achieves the proof. □

4 Infinitely many solutions

In this section, we prove under some symmetry condition on the function F that the problem (1) possesses infinitely many nontrivial weak solutions. The proof is based on Bartsch's fountain theorem [1]. Since $W_0^{1,p(x)}$ and $W_0^{1,q(x)}$ are reflexive and

separable (and their dual), then W and W^* are too. Let $(e_i)_{i\in\mathbb{N}} \subset W$ and $(e_i^*)_{i\in\mathbb{N}} \subset W^*$ such that

$$W = \overline{\text{span}\{e_i \,:\, i \in \mathbb{N}\}}, \quad W^* = \overline{\text{span}\{e_i^* \,:\, i \in \mathbb{N}\}}$$

and

$$< e_i^*, e_j >= \delta_{ij}, \quad \forall\, i, j \,\in \mathbb{N},$$

where $<\ .\,,\,.\ >$ denotes the duality product between W^* and W, and δ_{ij} is the Kronecker symbol. We define $X_j := \mathbb{R}e_j$, $Y_k := \oplus_{j=0}^{k} X_j$ and $Z_k := \oplus_{j=k}^{\infty} X_j$.
Let us recall the version of the Fountain theorem which will be used in the sequel.

Theorem 6 *Let $J \in C^1(W, \mathbb{R})$ be an even functional, where $(W, ||\,.\,||)$ is a separable and reflexive Banach space. If for every $k \in \mathbb{N}$, there are $0 < r_k < \rho_k$ such that*

(i) $\max\{J(u) \,:\, u \in Y_k,\ ||u|| = \rho_k\} \leq 0$,

(ii) $\inf\{J(u) \,:\, u \in Z_k,\ ||u|| = r_k\} \longrightarrow +\infty$ *as* $k \longrightarrow +\infty$,

(iii) J satisfies the Palais-Smale condition for every level,

then J has an unbounded sequence of critical points.

For every $a > 1$, $u \in L^a(\Omega)$ and $v \in L^a(\Omega)$, we define

$$|(u, v)|_a := \max\{|u|_a, |v|_a\}.$$

In (H_1), let $\widetilde{\alpha}$, $\widetilde{\beta}$ be two continuous and positive functions on $\overline{\Omega}$ such that

$$\frac{\alpha(x) + \widetilde{\alpha}(x)}{p^*(x)} + \frac{\beta(x) + \widetilde{\beta}(x)}{q^*(x)} = 1, \quad \forall\, x \in \overline{\Omega}.$$

Set

$$a := \max_{x\in\overline{\Omega}} \left\{ \frac{\alpha(x) + \widetilde{\alpha}(x)}{p^*(x)}, \frac{\beta(x) + \widetilde{\beta}(x)}{q^*(x)}, p_1(x), q_1(x) \right\},$$

$$b := \min_{x\in\overline{\Omega}} \left\{ \frac{\alpha(x) + \widetilde{\alpha}(x)}{p^*(x)}, \frac{\beta(x) + \widetilde{\beta}(x)}{q^*(x)}, p_1(x), q_1(x) \right\}.$$

Then we have the following

Lemma 7 *Define*

$$\beta_k := \sup\{|(u, v)|_a \,:\, ||(u, v)|| = 1,\ (u, v) \in Z_k\}.$$

Then, $\lim\limits_{k\to+\infty} \beta_k = 0$.

PROOF. It is clear that the sequence (β_k) is nonincreasing and positive. Let $\ell \geq 0$ such that $\lim\limits_{k\to+\infty} \beta_k = \ell$, and $(u_k, v_k) \in Z_k$ with $||(u_k, v_k)|| = 1$, $0 \leq \ell - |(u_k, v_k)|_a \leq 1/k$. Passing if necessary to a subsequence, there is $(u, v) \in W$ such that (u_k, v_k) converges weakly to (u, v) in W. On the other hand, for every $j \in \mathbb{N}$,

$$< e_j^*, (u, v) >= \lim_{k\to+\infty} < e_j^*, (u_k, v_k) >= 0.$$

Therefore, $(u, v) = 0$ and u_k (resp. v_k) converges weakly to u (resp. v) in $W_0^{1,p(x)}(\Omega)$ (resp. $W_0^{1,q(x)}(\Omega)$). By vertue of the compact embeddings $W_0^{1,p(x)}(\Omega) \subset L^a(\Omega)$, $W_0^{1,q(x)}(\Omega) \subset L^a(\Omega)$, it follows that (u_k, v_k) converges strongly to 0 in W and finally that $\ell = 0$. $\square$

In the following theorem, we show the existence of infinitely many solutions to the problem (1) under some symmetry assumption on F. We confine ourselves to the case where $p(x) = q(x)$ for every $x \in \overline{\Omega}$. Notice that the result remains valid for $p \neq q$ by adding some slight (and technical) changes in the hypothese (H_2).

Theorem 8 *If $F(x,u,v)$ is even in u, v and satisfies the hypotheses (H_1) and (H_2), then (1) possesses infinitely many (pairs) of solutions with unbounded energy.*

PROOF. It suffices to show that J has an unbounded sequence of critical points. The proof is based on the fountain theorem (see [1]). Let $(u_k, v_k) \in Z_k$ such that $||(u_k, v_k)|| = r_k \geq 1$ (r_k will be specified below). We may assume, without loss of generality, that $||(u_k, v_k)|| = ||u_k||_p$. In what follows, we will use the mean value theorem in the following form: For every $\gamma \in C_+(\overline{\Omega})$, $u \in L^{\gamma}(\Omega)$, there is $\xi \in \Omega$ such that

$$\int_\Omega |u|^{\gamma(x)}\, dx = |u|_\gamma^{\gamma(\xi)}.$$

Indeed, it is well known that there is $\xi \in \Omega$ such that

$$1 = \int_\Omega \left(|u|/|u|_\gamma\right)^{\gamma(x)}\, dx = \int_\Omega |u|^{\gamma(x)}\, dx / |u|_\gamma^{\gamma(\xi)},$$

and the claim is complete. It follows,

$$\begin{aligned}
J(u_k, v_k) &\geq \frac{1}{p^+}||u_k||_p^{p^-} - c\int_\Omega \left(1 + |u_k|^{p_1(x)} + |v_k|^{q_1(x)} + |u_k|^{\alpha(x)}|v_k|^{\beta(x)}\right) dx \\
&\geq \frac{1}{p^+}||u_k||_p^{p^-} - c|u_k|_{p_1}^{p_1(\xi_1^k)} - c|v_k|_{q_1}^{q_1(\xi_2^k)} - \\
&\quad -c|u_k|_{p^*\alpha/(\alpha+\tilde{\alpha})}^{p^*\alpha/(\alpha+\tilde{\alpha})(\eta_1^k)} - c|v_k|_{p^*\beta/(\beta+\tilde{\beta})}^{p^*\beta/(\beta+\tilde{\beta})(\eta_2^k)} - c|\Omega|,
\end{aligned}$$

where ξ_1^k, ξ_2^k, η_1^k, $\eta_2^k \in \Omega$ and $|\Omega|$ denotes the measure of Ω. Therefore,

$$\begin{aligned}
J(u_k, v_k) &\geq \frac{1}{p^+}||u_k||_p^{p^-} - c_1|u_k|_a^{p_1(\xi_1^k)} - c_1|v_k|_a^{q_1(\xi_2^k)} - \\
&\quad -c_1|u_k|_a^{p^*\alpha/(\alpha+\tilde{\alpha})(\eta_1^k)} - c_1|v_k|_a^{p^*\beta/(\beta+\tilde{\beta})(\eta_2^k)} - c|\Omega|, \\
&\geq \frac{1}{p^+}||u_k||_p^{p^-} - c_1(\beta_k||u_k||)^{p_1(\xi_1^k)} - c_1(\beta_k||u_k||)^{q_1(\xi_2^k)} - \\
&\quad -c_1(\beta_k||u_k||)^{p^*\alpha/(\alpha+\tilde{\alpha})(\eta_1^k)} - c_1(\beta_k||u_k||)^{p^*\beta/(\beta+\tilde{\beta})(\eta_2^k)} - c|\Omega| \\
&\geq \frac{1}{p^+}||u_k||_p^{p^-} - c_2\beta_k^b||u_k||^a - c|\Omega|,
\end{aligned}$$

where a, b are defined above. At this stage, we fix r_k as follows

$$r_k := \left(\frac{\beta_k^{-b}}{2c_2p^+}\right)^{\frac{1}{a-p^-}} \longrightarrow +\infty \quad \text{as} \quad k \longrightarrow +\infty.$$

Consequently, if $||(u_k, v_k)|| = r_k$ then

$$J(u_k, v_k) \geq \frac{1}{2p^+}||u_k||_p^{p^-} - c|\Omega|$$
$$= \frac{1}{2p^+}||(u_k, v_k)||^{p^-} - c|\Omega|.$$

On the other hand, it is known from (H_2) that $F(x,u,v) \geq c\left(|u|^{\theta_1} + |v|^{\theta_2} - 1\right)$, for every $x \in \Omega$ and $u,\ v \in \mathbb{R}$. Whence, if $(u,v) \in Y_k$ with $u \neq 0$, $v \neq 0$, we get from above that

$$\lim_{t \to +\infty} J(tu, tv) = -\infty.$$

This implies that

$$\max\{J(u,v) \,:\, ||(u,v)|| = \rho_k,\ (u,v) \in Y_k\} \leq 0$$

for every ρ_k large enough. Applying the fountain theorem, we achieve the proof. □

Acknowledgments

I am very grateful to Prof. Xian-Ling Fan for sending me generously many of his articles on this subject.

References

[1] T. Bartsch, Infinitely many solutions of a symmetric Dirichlet problem. Nonlinear Anal. **20**, 1205-1216 (1993)

[2] D. G. De Figueiredo, Semilinear elliptic systems: a survey of superlinear problems. Resenhas **2**, 373-391 (1996)

[3] D. E. Edmunds and J. Rákosník, Sobolev embeddings with variable exponent. Studia Math. **143**, 267-293 (2000)

[4] X. L. Fan, Regularity of nonstandard Lagrangians $f(x,\xi)$. Nonlinear Anal. **27**, 669-678 (1996)

[5] X. L. Fan and D. Zhao, A class of De Giorgi type and Hölder continuity. Nonlinear Anal. **6**, 295-318 (1999)

[6] X. L. Fan and D. Zhao, The quasi-minimizer of integral functionals with $m(x)$ growth conditions. Nonlinear Anal. **39**, 807–816 (2000)

[7] X. L. Fan, Y. Zhao and D. Zhao, Compact imbedding theorems with symmetry of Strauss-Lions type for the space $W^{1,p(x)}(\Omega)$. J. Math. Anal. Appl. **255**, 333–348 (2001)

[8] X. L. Fan, J. Shen and D. Zhao, Sobolev embedding theorems for spaces $W^{k,p(x)}(\Omega)$. J. Math. Anal. Appl. **262**, 749–760 (2001)

[9] X. L. Fan and D. Zhao, On the spaces $L^{p(x)}(\Omega)$ and $W^{m,p(x)}(\Omega)$. J. Math. Anal. Appl. **263**, 424–446 (2001)

[10] X. L. Fan and Q. H. Zhang, Existence of solutions for $p(x)$-Laplacian Dirichlet problem. Nonlinear Anal. **52**, 1843–1852 (2003)

[11] P. Marcellini, Regularity of minimizers of integrals of the calculus of variations with nonstandard growth conditions. Arch. Rational Mech. Anal. **105**, 267-284 (1989)

[12] P. Marcellini, Regularity and existence of solutions of elliptic equations with p, q-growth conditions. J. Differential Equations **90**, 1-30 (1991)

[13] J. Mawhin and M. Willem, Critical Point Theory and Hamiltonian Systems. Springer Verlag, Berlin (1989)

[14] J. Musielak, Orlicz spaces and modular spaces. Lecture Notes in Mathematics, 1034. Springer-Verlag, Berlin, (1983)

[15] M. Ruzicka, Electrorheological fluids: modeling and mathematical theory. Lecture Notes in Mathematics, 1748. Springer-Verlag, Berlin, (2000)

[16] M. Ruzicka, Flow of shear dependent electrorheological fluids. C. R. Acad. Sci. Paris Sér. I Math. **329**, 393-398 (1999) 393–398.

[17] M. Struwe, Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems. Springer-Verlag, (1996)

[18] M. Willem, Minimax theorems. Birkhäuser, Basel, (1996)

[19] V. V. Zhikov, Averaging of functionals of the calculus of variations and elasticity theory Math. USSR-Izv. **29**, 33–66 (1987)

Liste des prépublications

99-1 Monique Jeanblanc et Nicolas Privault. A complete market model with Poisson and Brownian components. A paraître dans *Proceedings of the Seminar on Stochastic Analysis, Random Fields and Applications*, Ascona, 1999.

99-2 Laurence Cherfils et Alain Miranville. Generalized Cahn-Hilliard equations with a logarithmic free energy. A paraître dans *Revista de la Real Academia de Ciencias.*

99-3 Jean-Jacques Prat et Nicolas Privault. Explicit stochastic analysis of Brownian motion and point measures on Riemannian manifolds. *Journal of Functional Analysis* **167** (1999) 201-242.

99-4 Changgui Zhang. Sur la fonction q-Gamma de Jackson. A paraître dans *Aequationes Math.*

99-5 Nicolas Privault. A characterization of grand canonical Gibbs measures by duality. A paraître dans *Potential Analysis.*

99-6 Guy Wallet. La variété des équations surstables. A paraître dans *Bulletin de la Société Mathématique de France.*

99-7 Nicolas Privault et Jiang-Lun Wu. Poisson stochastic integration in Hilbert spaces. *Annales Mathématiques Blaise Pascal*, **6** (1999) 41-61.

99-8 Augustin Fruchard et Reinhard Schäfke. Sursabilité et résonance.

99-9 Nicolas Privault. Connections and curvature in the Riemannian geometry of configuration spaces. *C. R. Acad. Sci. Paris, Série I* **330** (2000) 899-904.

99-10 Fabienne Marotte et Changgui Zhang. Multisommabilité des séries entières solutions formelles d'une équation aux q-différences linéaire analytique. A paraître dans *Annales de l'Institut Fourier*, 2000.

99-11 Knut Aase, Bernt Øksendal, Nicolas Privault et Jan Ubøe. White noise generalizations of the Clark-Haussmann-Ocone theorem with application to mathematical finance. *Finance and Stochastics*, **4** (2000) 465-496.

00-01 Eric Benoît. Canards en un point pseudo-singulier nœud. A paraître dans *Bulletin de la Société Mathématique de France.*

00-02 Nicolas Privault. Hypothesis testing and Skorokhod stochastic integration. *Journal of Applied Probability*, **37** (2000) 560-574.

00-03 Changgui Zhang. La fonction théta de Jacobi et la sommabilité des séries entières q-Gevrey, I. *C. R. Acad. Sci. Paris, Série I* **331** (2000) 31-34.

00-04 Guy Wallet. Déformation topologique par changement d'échelle.

00-05 Nicolas Privault. Quantum stochastic calculus for the uniform measure and Boolean convolution. A paraître dans *Séminaire de Probabilités* **XXXV**.

00-06 Changgui Zhang. Sur les fonctions q-Bessel de Jackson.

00-07 Laure Coutin, David Nualart et Ciprian A. Tudor. Tanaka formula for the fractional Brownian motion. A paraître dans *Stochastic Processes and their Applications.*

00-08 Nicolas Privault. On logarithmic Sobolev inequalities for normal martingales. *Annales de la Faculté des Sciences de Toulouse* **9** (2000) 509-518.

01-01 Emanuelle Augeraud-Veron et Laurent Augier. Stabilizing endogenous fluctuations by fiscal policies; Global analysis on piecewise continuous dynamical systems. A paraître dans *Studies in Nonlinear Dynamics and Econometrics*

01-02 Delphine Boucher. About the polynomial solutions of homogeneous linear differential equations depending on parameters. A paraître dans *Proceedings of the 1999 International Symposium on Symbolic and Algebraic Computation: ISSAC 99, Sam Dooley Ed., ACM, New York 1999.*

01-03 Nicolas Privault. Quasi-invariance for Lévy processes under anticipating shifts.

01-04 Nicolas Privault. Distribution-valued iterated gradient and chaotic decompositions of Poisson jump times functionals.

01-05 Christian Houdré et Nicolas Privault. Deviation inequalities: an approach via covariance representations.

01-06 Abdallah El Hamidi. Remarques sur les sentinelles pour les systèmes distribués

02-01 Eric Benoît, Abdallah El Hamidi et Augustin Fruchard. On combined asymptotic expansions in singular perturbation.

02-02 Rachid Bebbouchi et Eric Benoît. Equations différentielles et familles bien nées de courbes planes.

02-03 Abdallah El Hamidi et Gennady G. Laptev. Nonexistence of solutions to systems of higher-order semilinear inequalities in cone-like domains.

02-04 Hassan Lakhel, Youssef Ouknine, et Ciprian A. Tudor. Besov regularity for the indefinite Skorohod integral with respect to the fractional Brownian motion: the singular case.

02-05 Nicolas Privault et Jean-Claude Zambrini. Markovian bridges and reversible diffusions with jumps.

02-06 Abdallah El Hamidi et Gennady G. Laptev. Existence and Nonexistence Results for Reaction-Diffusion Equations in Product of Cones.

02-07 Guy Wallet. Nonstandard generic points.

02-08 Gilles Bailly-Maitre. On the monodromy representation of polynomials.

02-09 Abdallah El Hamidi. Necessary conditions for local and global solvability of nondiagonal degenerate systems.

02-10 Abdallah El Hamidi et Amira Obeid. Systems of Semilinear higher order evolution inequalities on the Heisenberg group.

03-01 Abdallah El Hamidi et Gennady G. Laptev. Non existence de solutions d'inéquations semilinéaires dans des domaines coniques.

03-02 Eris Benoît et Marie-Joëlle Rochet. A continuous model of biomass size spectra governed by predation and the effects of fishing on them.

03-03 Catherine Stenger: On a conjecture of Wolfgang Wasow concerning the nature of turning points.

03-04 Christian Houdré et Nicolas Privault. Surface measures and related functional inequalities on configuration spaces.

03-05 Abdallah El Hamidi et Mokhtar Kirane. Nonexistence results of solutions to systems of semilinear differential inequalities on the Heisenberg group.

03-06 Uwe Franz, Nicolas Privault et René Schott. Non-Gaussian Malliavin calculus on real Lie algebras.

04-01 Abdallah El Hamidi. Multiple solutions to a nonlinear elliptic equation involving Paneitz type operators.

04-02 Mohamed Amara, Amira Obeid et Guy Vallet. Relaxed formulation and existence result of the degenerated elliptic small disturbance model.

04-03 Hippolyte d'Albis et Emmanuelle Augeraud-Veron. Competitive Growth in a Life-cycle Model: Existence and Dynamics

04-04 Sadjia Aït-Mokhtar: Third order differential equations with fixed critical points.

04-05 Mokhtar Kirane et Nasser-eddine Tatar. Asymptotic Behavior for a Reaction Diffusion System with Unbounded Coefficients.

04-06 Mokhtar Kirane, Eric Nabana et Stanislav I. Pohozaev. Nonexistence of Global Solutions to an Elliptic Equation with a Dynamical Boundary Condition.

04-07 Khaled M. Furati, Nasser-eddine Tatar and Mokhtar Kirane. Existence and asymptotic behavior for a convection Problem.

04-08 José Alfredo López-Mimbela et Nicolas Privault. Blow-up and stability of semilinear PDE's with gamma generator.

04-09 Abdallah El Hamidi. Multiple solutions with changing sign energy to a nonlinear elliptic equation.

04-10 Sadjia Aït-Mokhtar: A singularly perturbed Riccati equation.
04-11 Mohamed Amara, Amira Obeid et Guy Vallet. Weighted Sobolev spaces for a degenerated nonlinear elliptic equation.
04-12 Abdallah El Hamidi. Existence results to elliptic systems with nonstandard growth conditions.