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Nonexistence of Global Solutions to a Hyperbolic Equation with a Space-Time Fractional Damping

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Abstract

We establish conditions that ensure the absence of global solutions to the nonlinear hyperbolic equation with a time-space fractional damping:

$$u_{tt} - \Delta u + (-\Delta)^{\beta/2} D_+^\alpha u = |u|^p,$$

where $(-\Delta)^{\beta/2}$, $1 \le \beta \le 2$ stands for the $\beta/2$ fractional power of the Laplacien and D_+^α is the Riemann-Liouville's time fractional derivative. Our results include nonexistence results as well as necessary conditions for the local and global solvability. The method used is based on a duality argument with an appropriate choice of the test function and a scaling argument.

Keywords: Hyperbolic equation; space-time fractional damping; Nonexistence.

1. Introduction

In this paper, we discuss the nonexistence of weak solutions to the nonlinear wave equation with a time-space fractional damping:

$$u_{tt} - \Delta u + (-\Delta)^{\beta/2} D_+^\alpha u = |u|^p, \tag{1}$$

posed in $Q_T = \mathbb{R}^N \times (0,T)$, $0 < T \le +\infty$, subject to the initial conditions:

$$u(0,x) = u_0(x), \;\; u_t(x,0) = u_1(x), x \in \mathbb{R}^N, \tag{2}$$

where $\Delta = \frac{\partial^2}{\partial x_1^2} + ... + \frac{\partial^2}{\partial x_N^2}$ is the usual laplacien in the space variable x, u_t is the time derivative of u, $(-\Delta)^{\beta/2}$ is the $\beta/2$ fractional power of the laplacien ($0 < \beta \le 2$) which stands for propagation in media with impurities and is defined by $(-\Delta)^{\beta/2} v(x) = \mathcal{F}^{-1}(|\xi|^\beta \mathcal{F}(v)(\xi))(x)$, where $\mathcal{F}$ denotes the Fourier transform and

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$\mathcal{F}^{-1}$ denotes its inverse; D^α_+ is the Riemann-Liouville fractional derivative defined by:

$$D^\alpha_+ u(x,t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{u(x,\sigma)}{(t-\sigma)^\alpha}\, d\sigma, \tag{3}$$

with the parameter $0 < \alpha \leq 1$.

Before we state our results, let us dwell on existing literature concerning equations close to Eq. (1) .

The time fractional derivative has long been found to be a very effective means to describe the anomalous attenuation behaviors. For example, Hanyga and Seredynska [11] studied the ordinary differential equation

$$D^2 u + \gamma D^{1+\eta} u + F(u) = 0, \tag{4}$$

where $D^{1+\eta}$, $0 < \eta < 1$, represents the $(1+\eta)$-order fractional derivative in the sense of Caputo which models the anomalous attenuation, and γ is the thermoviscous coefficient. Recently, Chen and Holm [4] studied the equation

$$\Delta v = \frac{1}{c_0^2} v_{tt} + \frac{2\alpha_0}{c_0^{1-\gamma}} (-\Delta)^{y/2} v_t, \tag{5}$$

which governs the propagation of sound through a viscous fluid, c_0 is the inviscid phase velocity, $2\alpha_0$ is the collective thermoviscous coefficient.

In [5], they extended their study to the wave equation model for frequency dependent lossy media

$$\Delta P = \frac{1}{c_0^2} P_{tt} + \gamma \frac{\partial^\eta}{\partial t^\eta} (-\Delta)^{s/2} P, \tag{6}$$

$0 \leq s \leq 2$, $0 < \eta \leq 3, \eta \neq 2$, where γ is a viscous constant, s and η can be arbitrary real numbers within their range of specification. Equations (5) and (6) can be seen as a generalization of the earlier important work of Greenberg, Mac Camy and Mizel [6] who considered the equation

$$\rho_0 u_{tt} = u_{xx} + \lambda u_{xtx} + g(x,t)$$

with $x \in \mathbb{R}, t > 0$, and ρ_0, λ are some constants that characterize the medium; $g(x,t)$ is a given function representing an external force.

As equations (5) and (6) may be viewed as approximations of nonlinear equations, Eq. (1) contains a nonlinear term that is a prototype of nonlinearities that may occur in practice.

Let's note also that in [3], Cholewa and Carvalho dealt with the equation

$$u_{tt} = \Delta u + (-\Delta)^\alpha\, u_t + |u|^p$$

which is Eq. (1) when $\alpha = 1$.

If $\beta = 0$ and $\alpha = 1$ in (1), then we obtain the wave equation with the linear damping u_t. In this case, Todorova and Yordanov [14], Kirane and Qafsaoui [9], and Zang [15] showed that the Fujita exponent is equal to $1 + 2/N$ which is the one of the heat equation $u_t - \Delta u = |u|^p$. The very interesting article of Todorova and Yordanov is in fact a "complete" study of Eq. (1) when $\beta = 0$ and $\alpha = 1$. However, when $\beta = 0$, $\alpha \neq 0$, Kirane and Tatar [8] and then Tatar [12] , [13] showed that under certain conditions, solutions may be exponentially unbounded in the L^p-norm for sufficiently large initial data. They mainly use the Fourier transform and the Hardy-Littlewood-Sobolev inequality.

It is the purpose of this article to provide:

1. the Fujita's exponent for Equation (1);

2. necessary conditions for the local and global non solvability of Equation (1). These reveals the effect of the behavior of the initial data at infinity on the local or global character of weak solutions.

The method we will use is based on the articles of Baras and Pierre [1], Baras and Kersner [2] and Kalashnikov [7] .

2. Mathematical background and results

Solutions to problem (1)-(2) are meant in the following sense.

Definition 1.1 A function $u \in L^1_{loc}(Q)$ is a local weak solution to (1) subject to (2) defined on Q, $0 < T < +\infty$, if $u \in L^p_{loc}(Q)$ is such that

$$\int_{\mathbb{R}^N} u_0(x)\varphi_t(x,0)dx - \int_{\mathbb{R}^N} u_1(x)\varphi(x,0)dx + \int_Q u\,\varphi_{tt}\,dxdt =$$

$$\int_Q u\,\Delta\varphi\,dxdt + \int_Q u\,(-\Delta)^{\beta/2} D^\alpha_- \varphi\,dxdt + \int_Q |u|^p\varphi\,dxdt$$

for any test function $\varphi \in C_0^\infty(Q)$, $\varphi \geq 0$.

The presence of the time-fractional derivative $D^\alpha_-\varphi$ in the definition is due to the formula of integration by parts for fractional derivative; in fact, if

$$(D^\alpha_+ f)(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{f(y)}{(t-y)^\alpha}\,dy, \quad 0 < \alpha < 1,$$

$$(D^\alpha_- f)(t) = -\frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_t^T \frac{f(y)}{(y-t)^\alpha}dy, \quad 0 < \alpha < 1,$$

then

$$\int_0^t f(x)\,(D^\alpha_- g)(x)dx = \int_0^T (D^\alpha_+ f)(x)\,g(x)dx.$$

Now, we are in position to announce our results.

Theorem. Let $p > 1$. Assume that

$$\liminf_{|x| \to +\infty} u_1(x) = +\infty,$$

then problem (1), (2) does not admit a weak local solution for any $T > 0$.

As it will be clear from the proof, $u(x,0)$ will play no important role; so it may be taken equal to zero. In this case, the Riemann-Liouville's time-fractional derivative will coincide with Caputo's one.

Proof. For any $0 \leq \varphi \in C_0^\infty(\mathbb{R}^N \times (0,T))$ such that

$supp\, \varphi \subset \left\{ x \in \mathbb{R}^N / |x| \geq R_0 > 0 \right\}$ ($supp$ stands for support), we have:

$$\begin{aligned} & \int_{\mathbb{R}^N} u_1(x)\varphi(x,0)\, dx + \int_Q |u|^p \varphi \, dxdt \\ \leq & \int_Q |u|\, |\Delta\varphi|\, dxdt + \int_Q |u|\, |\varphi_{tt}|\, dxdt + \int_Q |u|\, |(-\Delta)^{\beta/2} D_-^\alpha \varphi|\, dxdt \end{aligned}$$

By the ε-Young's inequality, we may estimate

$$\int_Q |u|\, |\varphi_{tt}|\, dxdt = \int_Q |u|\, \varphi^{1/p}\, |\varphi_{tt}|\, \varphi^{-1/p} \leq \varepsilon \int_Q |u|^p\, \varphi\, dxdt + C_\varepsilon \int_Q |\varphi_{tt}|^{p'}\, \varphi^{-p'/p}\, dxdt,$$

where $p + p' = pp'$, $\varepsilon > 0; C_\varepsilon$ is a constant depending only on ε.

Similarly, we have

$$\int_Q |u|\, |\Delta\varphi|\, dxdt \leq \varepsilon \int_Q |u|^p\, \varphi\, dxdt + C_\varepsilon \int_Q |\Delta\varphi|^{p'} \varphi^{-p'/p}\, dxdt$$

and

$$\begin{aligned} & \int_Q |u|\, |(-\Delta)^{\beta/2} D_-^\alpha \varphi|\, dxdt \\ \leq & \ \varepsilon \int_Q |u|^p\, \varphi\, dxdt + C_\varepsilon \int_Q |(-\Delta)^{\beta/2} D_-^\alpha \varphi|^{p'} \varphi^{-p'/p}\, dxdt. \end{aligned}$$

So if $2\varepsilon = 1$, we then clearly have

$$I = \int_{\mathbb{R}^N} u_1(x)\varphi(x,0)\, dx \leq C\Big(\mathcal{A} + \mathcal{B} + \mathcal{C}\Big)$$

for some constant C, where

$$\begin{aligned}
\mathcal{A} \ : \ &= \int_Q |\varphi_{tt}|^{p'} \varphi^{-p'/p} \, dxdt, \\
\mathcal{B} \ : \ &= \int_Q |\Delta\varphi|^{p'} \varphi^{-p'/p} \, dxdt, \\
\mathcal{C} \ : \ &= \int_Q |(-\Delta)^{\beta/2} D_-^{\alpha}\varphi|^{p'} \varphi^{-p'/p} \, dxdt.
\end{aligned}$$

At this stage, we make the judicious choice

$$\varphi(x,t) = \Phi\Big(\frac{x}{R}\Big)\Big(1 - \frac{t^2}{T^2}\Big)^{2p'}$$

where $\Phi \in W^{1,\infty}(\mathbb{R}^N)$, $\Phi \geq 0$, $supp\Phi \subset \{|x| \leq 2\}$ is such that

$$|\Delta\Phi| \leq k\Phi.$$

It is clear, from our choice of φ that the requirements

$$\varphi(x,T) = \varphi_t(x,T) = \varphi_t(x,0) = 0$$

are satisfied.

Now, we estimate $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ in terms of T and R.

First, if we set $t = \tau T$, $\gamma = p'(p'-1)$, $\mathcal{A}$ will be written

$$\mathcal{A} = \gamma^{p'} T^{1-2p'} \int_{Q_1} \Phi \, dxd\tau$$

Second, using $|\Delta\Phi| \leq k\Phi$, we obtain

$$\mathcal{B} \leq Tk^{p'} R^{-2p'} \int_{Q_1} \Phi \, dxd\tau$$

(we have changed x/R into x).

Third, we can write

$$\mathcal{C} = TR^{-pp'} \Lambda^{p'}_{\alpha,p'} T^{-4p'^2} \int_{Q_1} \Phi \, dxd\tau,$$

where

$$\Lambda^{p'}_{\alpha,p'} = \frac{(4p'-\alpha)}{2} \frac{\Gamma(\frac{1-\alpha}{2})}{\Gamma(1-\alpha)} \frac{\Gamma(1+2p')}{\Gamma(2p'+\frac{3-\alpha}{2})}.$$

Now, observe that

$$\begin{aligned}
\inf_{|x|>R} u_1(x) \int_{\mathbb{R}^N} \Phi(x) \, dx \ &\leq \ \int_{\mathbb{R}^N} u_1 \Phi(x) dx \leq \qquad (7) \\
&\Big\{\gamma^{p'} T^{2-2p'} + T^2 k^{p'} R^{-2p'} + R^{-pp'} \Lambda^{p'}_{\alpha,p'} T^{2-4p'^2}\Big\} \int_{\mathbb{R}^N} \Phi(x) dx.
\end{aligned}$$

As $\int_{\mathbb{R}^N} \Phi(x)dx > 0$, we divide (7) by $\int_{\mathbb{R}^N} \Phi(x)dx$ and let R goes to infinity; we obtain:

$$\liminf_{|x|\to+\infty} u_1(x) \leq \gamma^{p'} T^{2-2p'}. \tag{8}$$

The estimate (8) leads to three consequences:

1. As $T < +\infty$, if $\liminf_{|x|\to+\infty} u_1(x) = +\infty$, we are lead to a contradiction.

2. The time of local existence can be estimated via (8) as follows:

$$T \leq \left(\gamma^{-p'} \liminf_{|x|\to+\infty} u_1(x)\right)^{-\frac{1}{2(p'-1)}}$$

3. The limit

$$\liminf_{|x|\to+\infty} u_1(x) = 0$$

is necessary for problem (1)-(2) to admit a global weak solution. But, as it can be seen, this condition is not sufficient.

Remark. Our analysis is certainly robust for more general nonlinear wave equations. It can surely be used, for example, for the following highly nonlinear equation

$$D_+^{\rho} u - \Delta |u|^{m-1} u + (-\Delta)^{\beta/2} D_+^{\alpha} |u|^{l-1} u = h(x,t)|u|^p + g(x,t),$$

where $1 < p$, $1 < \rho \leq 2$, $1 \leq m$, $1 \leq \beta \leq 2, 0 < \alpha < 1$ and $h(x,t) \geq t^{\sigma}|u|^{\gamma}$ for $|x| >> 1$. This equation interpolates the heat equation and the wave equation.

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