Critical Exponents of Fujita Type for Certain Evolution Equations and Systems with Spatio-Temporal Fractional Derivatives

Mokhtar Kirane, Yamina Laskri et Nasser-eddine Tatar

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M. Kirane

Laboratoire de Mathématiques, Pôle Sciences et Technologie, Université de La Rochelle, Avenue M. Crépeau, 17042 La Rochelle Cedex, France

E-mail: mkirane@univ-lr.fr

Y. Laskri

Faculté des Sciences, Département de Mathématiques, Université de Annaba, B. P. 12, 23000 Annaba, Algeria

E-mail: ylaskri@yahoo.fr

N.-e. Tatar

King Fahd University of Petroleum and Minerals, Department of Mathematical Sciences, P.O. Box 1446, Dhahran, 31261, Saudi Arabia

E-mail: tatarn@kfupm.edu.sa

Abstract: This paper is concerned with establishing necessary or sufficient conditions for the existence of solutions to evolution equations with fractional derivatives in space and time. The Fujita exponent is determined. Then, these results are extended to systems of reaction-diffusion equations. Our results are new and shed lights on important practical questions.

Key words: Fractional derivatives; Fujita's exponent; nonlinear evolution equations ; nonlinear reaction-diffusion systems; porous media.

1 INTRODUCTION

In this article, we are concerned with finding sufficient conditions and necessary conditions for the solvability of evolution equations and systems with temporal and spatial fractional derivatives. In the first part, attention is paid to the evolution problem **(STFE)**:

$$\begin{cases} \mathbf{D}_{0|t}^{\alpha}u + (-\Delta)^{\beta/2}(u) = h(x,t)|u|^{1+\widetilde{p}} & \text{for } (x,t) \in \mathbb{R}^N \times \mathbb{R}^+ =: Q \\ u(x,0) = u_0(x) \geq 0 & \text{for } x \in \mathbb{R}^N \end{cases}$$

where $\mathbf{D}_{0|t}^{\alpha}$ denotes the time-derivative of arbitrary order $\alpha \in (0,1)$ in the sense of Caputo [23], $(-\Delta)^{\beta/2}$, $\beta \in [1,2]$, is the $(\beta/2)$-fractional power of the Laplacian $-\Delta_x$ in the x variable; it is defined by:

$$(-\Delta)^{\beta/2}v(x,t) = \mathcal{F}^{-1}(|\xi|^\beta \mathcal{F}(v)(\xi))(x,t),$$

where $\mathcal{F}$ denotes the Fourier transform and $\mathcal{F}^{-1}$ its inverse; the function $h(x,t)$ will be specified later. The exponent $\widetilde{p}$ is strictly positive.

In first equation in **(STFE)**, $(-\Delta)^{\beta/2}$ is a multidimensional fractal (anomalous) diffusion related to the Lévy flights. The time fractional derivative $D_{0|t}^{\alpha}$ accounts also for dispersive anomalous diffusion, characterized by the mean square displacement $\langle x^2 \rangle \propto t^\alpha, 0 < \alpha < 1$. Dispersive anomalous diffusion can be derived from CTRW models [19], based on the assumption of random jump lengths and random waiting times between successive particle jumps. Important physical applications of CTRW models include diffusion of carriers in amorphous photo-conductors, diffusion in turbulent flow, a percolation model in porous media ($\alpha = 1/2$) [21], fractal media [26], various biological phenomena [5] and finance [28]. The contribution [4] is related to fluid dynamics.

In the case $\alpha = 1, \beta = 2$, the first equation in **(STFE)** reduces to the usual heat equation which is well documented.

In fact, in his pioneering article [8], Fujita considered the Cauchy problem **(FE)**

$$\begin{cases} u_t = \Delta u + |u|^{1+\widetilde{p}}, & \text{in } Q \\ u(x,0) = a(x) \geq 0, & \text{in } \mathbb{R}^N \end{cases} \tag{1}$$

where $0 < \widetilde{p}$. If $p_c := \frac{2}{N}$ (c for critical), he proved that:

(i) if $0 < \widetilde{p} < p_c$ and $a(x_0) > 0$ for some x_0, then any solution to **(FE)** blows-up in a finite time.

(ii) if $p > p_c$, then there exist solutions on Q as well as solutions which exist on $\mathbb{R}^N \times (0,T)$ for some finite T but not on Q. (For this p, not all solutions are global; indeed, if $(1/2) \int_{\mathbb{R}^N} |\nabla u_0|^2 \, dx - (1/(p+1)) \int_{\mathbb{R}^N} u_0^p \, dx < 0$, the solution cannot be

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global [16].)

The critical case $p = p_c$ was decided later by Hayakawa [11] for $N = 1, 2$, and by Kobayashi, Sirao and Tanaka [14] for $N \geq 3$.

Later on Nagasawa and Sirao [20], Sujitani [31], and Guedda and Kirane [9] considered the problem:

$$\begin{cases} u_t + (-\Delta)^{\beta/2}(u) = c(x,t)|u|^{1+\widetilde{p}}, & \text{in } Q \\ u(x,0) = u_0(x) \geq 0, & \text{in } \mathbb{R}^N. \end{cases}$$

Nagasawa and Sirao have taken $c(x,t) = c(x)$, Sujitani $c(x,t) = 1$, while Guedda and Kirane [9] studied the case $c(x,t) = c(t)$. The method of proof in [20] is probabilistic while in [31] and [9], the approach is analytic.

In a more recent article, Guedda and Kirane [10] extended the previous results to the equation:

$$u_t + (-\Delta)^{\beta/2}(u) = h(x,t)|u|^{1+p}, \qquad \text{in } Q,$$

where $h(x,t) = O(t^\sigma |x|^\rho)$ for large $|x|$.

Finally, Kirane and Qafsaoui [13] treated the more general equation:

$$u_t + (-\Delta)^{\beta/2}(u^m) + a(x,t) \cdot \nabla u^q = f(x,t)|u|^{1+p}, \quad \text{in } Q,$$

which covers in particular the equation considered by Qi [32]

$$u_t - \Delta(u^m) = |x|^\sigma t^s |u|^{1+\widetilde{p}}, \quad \text{in } Q.$$

The, here above, cited articles follow either Fujita's article or the duality argument with a nonlinear capacity estimate. In a special situation, this has been used by Baras and Pierre [2]. A more versatile variant has been introduced by Mitidieri and Pohozaev [18], Pohozaev and Tesei [24], and then used by Guedda and Kirane [10], Laptev [17] and Kuiper [15] (to cite but a few).

To ensure that the problem **(STFE)** is well posed, the fractional derivative has to be interpreted in the Caputo sense [23] (also cf. [29] for a justification of the choice of Caputo derivatives for a nonlinear ordinary differential equation with fractional derivatives).

Our theorems are reduced to the assertion on the nonexistence of solutions. If an existence result of solutions to the Cauchy problem holds, then the nonexistence of solutions means that every nonnegative solution blows-up in finite time.

To have an idea about ill-posed problems, one is referred to the important contributions [6], [22], [27] and [30].

We recall here some definitions of fractional derivatives.

The left-handed derivative and the right-handed derivative in the Riemann-Liouville sense for $\Psi \in L^1(0,T)$, $0 < \alpha < 1$ are defined as follows:

$$(D_{0|t}^\alpha \Psi)(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t \frac{\Psi(\sigma)}{(t-\sigma)^\alpha} \, d\sigma,$$

where the symbol Γ stands for the usual Euler gamma function, and

$$(D_{t|T}^\alpha \Psi)(t) = -\frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_t^T \frac{\Psi(\sigma)}{(\sigma-t)^\alpha} \, d\sigma,$$

respectively.

The Caputo derivative

$$(\mathbf{D}_{0|t}^\alpha \Psi)(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{\Psi'(\sigma)}{(t-\sigma)^\alpha} \, d\sigma,$$

requires $\Psi' \in L^1(0,T)$.
Clearly, we have

$$(D_{0|t}^\alpha g)(t) = \frac{1}{\Gamma(1-\alpha)} \left[\frac{g(0)}{t^\alpha} + \int_0^t \frac{g'(\sigma)}{(t-\sigma)^\alpha} \, d\sigma \right]$$

and

$$(D_{t|T}^\alpha f)(t) = \frac{1}{\Gamma(1-\alpha)} \left[\frac{f(T)}{(T-t)^\alpha} - \int_t^T \frac{f'(\sigma)}{(\sigma-t)^\alpha} \, d\sigma \right].$$

Therefore the Caputo derivative is related to the Riemann-Liouville derivative by

$$\mathbf{D}_{0|t}^\alpha \Psi(t) = D_{0|t}^\alpha [\Psi(t) - \Psi(0)].$$

We have the formula of integration by parts:

$$\int_0^T f(t) \, (D_{0|t}^\alpha g)(t) \, dt = \int_0^T (D_{t|T}^\alpha f)(t) \, g(t) \, dt.$$

Solutions to problem (**STFE**) are meant in the following sense.

Definition 1. Let $p = \widetilde{p} + 1$. A function $u \in L_{loc}^1(Q_T)$ ($Q_T := \mathbb{R}^N \times (0,T)$) is a local weak solution to (**STFE**) defined on Q_T, if $uh^{1/p} \in L_{loc}^p(Q_T, \, dxdt)$ and is such that

$$\int_{Q_T} u_0(x) D_{t|T}^\alpha \varphi(x,t) \, dxdt + \int_{Q_T} h \, |u|^p \varphi \, dxdt$$
$$= \int_{Q_T} u \, (-\Delta)^{\beta/2} \varphi \, dxdt + \int_{Q_T} u \, D_{t|T}^\alpha \varphi \, dxdt \tag{2}$$

for any test function $\varphi \in C_{x,t}^{2,1}(Q_T)$, such that $\varphi(x,T) = 0$.

The integrals in the definition are supposed to be convergent.
If in the above definition, $T = +\infty$, the solution is called global.

Concerning the function $h(x,t)$ we require the condition **(H)**

$$h(x,t) \geq C_h |x|^\sigma t^\rho \quad \text{for} \ \ x \in \mathbb{R}^N, \ t > 0, \ C_h > 0.$$

The assumptions on σ and ρ will be determined through the convergence of certain integrals in the proof (see (3) below). It can be easily seen that no conditions will be imposed on σ and ρ in case $t \geq t_0 > 0$, $|x| < R$ and h^{1-p} is integrable in a ball of radius R in x and radius t_0 in t.

1.1   THE RESULTS

Now, we are in position to announce our first result.

Theorem 1. *Let $N \geq 1$ and $p > 1$. Assume that (H) is satisfied. If*

$$1 < p \leq p_c = 1 + \frac{\alpha(\beta + \sigma) + \beta\rho}{\alpha N + \beta(1 - \alpha)},$$

*then problem **(STFE)** admits no global weak nonnegative solutions other than the trivial one.*

Proof.   The proof proceeds by contradiction. Suppose that u is a nontrivial nonnegative solution which exists globally in time. That is u exists in $(0, T^*)$ for any arbitrary $T^* > 0$. Let T and R be two positive real numbers such that $0 < TR^{\beta/\alpha} < T^*$.

For later use, let Φ be a smooth nonincreasing function such that

$$\Phi(z) = \begin{cases} 1 & \text{if} \ \ z \leq 1, \\ 0 & \text{if} \ \ z \geq 2, \end{cases}$$

and $0 \leq \Phi \leq 1$.

The test function φ is chosen so that

$$\int_{Q_T} |(-\Delta)^{\beta/2}\varphi|^{p'} (h\varphi)^{-p'/p} < \infty, \quad \int_{Q_T} |D^\alpha_{t|T}\varphi|^{p'} (h\varphi)^{-p'/p} < \infty. \tag{3}$$

To estimate the right hand side of (2) on $Q_{TR^{2/\theta}}$, we write

$$\int_{Q_{TR^{2/\theta}}} u(-\Delta)^{\beta/2}(\varphi) = \int_{Q_{TR2/\theta}} u \, (h\varphi)^{\frac{1}{p}} \, (-\Delta)^{\beta/2}(\varphi) \, (h\varphi)^{-\frac{1}{p}}.$$

Using the ε-Young inequality

$$XY \leq \varepsilon X^p + C(\varepsilon) Y^{p'}, \quad p + p' = pp', \quad X \geq 0, Y \geq 0,$$

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we have the estimate

$$\int_{Q_{TR^{2/\theta}}} u\,(-\Delta)^{\beta/2}\varphi \le \varepsilon \int_{Q_{TR^{2/\theta}}} |u|^p h\varphi + C(\varepsilon) \int_{Q_{TR^{2/\theta}}} |(-\Delta)^{\beta/2}\varphi|^{p'}(h\varphi)^{-p'/p}.$$

Similarly,

$$\int_{Q_{TR^{2/\theta}}} u\,D^{\alpha}_{t|TR^{2/\theta}}\varphi \le \varepsilon \int_{Q_{TR^{2/\theta}}} |u|^p h\varphi + C(\varepsilon) \int_{Q_{TR^{2/\theta}}} |D^{\alpha}_{t|TR^{2/\theta}}\varphi|^{p'}(h\varphi)^{-p'/p}.$$

Now, taking ε small enough, we obtain the estimate

$$\int_{Q_{TR^{2/\theta}}} h|u|^p\varphi \le C(\varepsilon) \int_{Q_{TR^{2/\theta}}} \left\{|(-\Delta)^{\beta/2}\varphi|^{p'} + |D^{\alpha}_{t|TR^{2/\theta}}\varphi|^{p'}\right\}(h\varphi)^{-p'/p}. \qquad (4)$$

At this stage, we set

$$\varphi(x,t) := \Phi\left(\frac{|x|^2 + t^\theta}{R^2}\right),$$

where R and θ are positive real numbers.

Let us perform the change of variables

$$\tau = t/R^{\frac{2}{\theta}}, \quad y = x/R,$$

and set

$$\Omega := \left\{(y,\,\tau) \in \mathbb{R}^N \times \mathbb{R}^+,\ |y|^2 + \tau^\theta < 2\right\}, \quad \mu(y,\tau) := \tau^\theta + |y|^2.$$

Now, we choose θ such that the right hand sides of

$$\int_{Q_{TR^{2/\theta}}} |(-\Delta)^{\beta/2}\varphi|^{p'}(h\varphi)^{-p'/p}$$

$$\le R^{-\beta p'+N+\frac{2}{\theta}-\frac{p'}{p}\left(\sigma+\frac{2\rho}{\theta}\right)} \int_\Omega |(-\Delta)^{\beta/2}\Phi \circ \mu|^{p'}(C_h\,|y|^\sigma\,\tau^\rho\,\Phi \circ \mu)^{-p'/p}\,dy d\tau$$

and

$$\int_{Q_{TR^{2/\theta}}} |D^{\alpha}_{t|TR^{2/\theta}}\varphi|^{p'}(h\varphi)^{-p'/p}$$

$$\le R^{-\frac{2}{\theta}\alpha p'+N+\frac{2}{\theta}-\frac{p'}{p}\left(\sigma+\frac{2\rho}{\theta}\right)} \int_\Omega |D^{\alpha}_{\tau|T}\Phi \circ \mu|^{p'}(C_h\,|y|^\sigma\,\tau^\rho\,\Phi \circ \mu)^{-p'/p}\,dy d\tau$$

are of the same order in R.

In doing so, we find $\theta = \frac{2\alpha}{\beta}$.

We then have the estimate

$$\int_{Q_{TR^{\beta/\alpha}}} h|u|^p\varphi \le CR^\gamma \qquad (5)$$

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where
$$\gamma = -\beta p' + N + \frac{\beta}{\alpha} - (\sigma + \frac{\rho\beta}{\alpha})\frac{p'}{p},$$
and
$$C = C(\varepsilon) \int_{\Omega} \left(|(-\Delta)^{\beta/2}\Phi \circ \mu|^{p'} + |D^{\alpha}_{\tau|T}\Phi \circ \mu|^{p'} \right) (C_h |y|^{\sigma}\tau^{\rho}\Phi \circ \mu)^{-p'/p} dy d\tau.$$

Now, if we choose $\gamma < 0$ (that is $p < p_c$) and let $R \to \infty$ in (5), we obtain
$$\int_{\mathbb{R}^N \times \mathbb{R}^+} h|u|^p \le 0. \tag{6}$$
this implies that $u = 0$ a.e., which is a contradiction.

In case $\gamma = 0$ (i.e. $p = p_c$), observe that (because of the convergence of the integral in (5)) if
$$\mathcal{C}_R = \left\{ (x,t) \in \mathbb{R}^N \times \mathbb{R}^+ : R^2 < |x|^2 + t^{\theta} \le 2R^2 \right\},$$
then
$$\lim_{R \to \infty} \int_{\mathcal{C}_R} |u|^p h\varphi \, dx dt = 0. \tag{7}$$
If instead of using the ε-Young inequality, we rather use the Hölder inequality, then instead of estimate (4), we find
$$\int_{Q_{TR^{\beta/\alpha}}} h|u|^p \varphi \, dx dt \le L \left(\int_{\mathcal{C}_R} |u|^p h\varphi \, dx dt \right)^{1/p} \tag{8}$$
where
$$L := \left\{ \left(\int_{\Omega_1} |D^{\alpha}_{\tau|T}\Phi \circ \mu|^{p'} (C_h |y|^{\sigma} \tau^{\rho} \Phi \circ \mu)^{-p'/p} \, dy d\tau \right)^{1/p'} \right.$$
$$\left. + \left(\int_{\Omega_1} |(-\Delta)^{\beta/2}\Phi \circ \mu|^{p'} (C_h |y|^{\sigma} \tau^{\rho} \Phi \circ \mu)^{-p'/p} \, dy d\tau \right)^{1/p'} \right\}$$
and
$$\Omega_1 = \left\{ (y,\tau) \in \mathbb{R}^N \times \mathbb{R}^+ : 1 \le |y|^2 + \tau^{\theta} \le 2 \right\}.$$
Using (8), we obtain via (7), after passing to the limit as $R \to \infty$
$$\int_{\mathbb{R}^N \times \mathbb{R}^+} |u|^p h \, dx dt = 0.$$
This leads to $u = 0$ a.e. and completes the proof.

Remark 1. *The requirement $\gamma \le 0$, i.e.,*
$$p \le 1 + \frac{\alpha(\beta + \sigma) + \beta\rho}{\alpha N + \beta(1 - \alpha)}$$
provides us with a critical exponent which coincides with the well known Fujita exponent in case $\sigma = \rho = 0$, $\alpha = 1$ and $\beta = 2$.

Remark 2. *The analysis could be performed for more general highly nonlinear equations such as*

$$D_{0|t}^{\alpha}(u - u_0) + (-\Delta)^{\beta/2}\left(|u|^{m-1}u\right) + a(x) \cdot \nabla\left(|u|^{q-1}u\right) = h(x,t)|u|^p.$$

It works also for other more general problems.

2 Systems of Fractional Differential Equations

In this section, we show how the method of proof used for the case of one equation can be carried out for the system of reaction-diffusion equations **(FDS)**

$$\begin{cases} D_{0|t}^{\alpha}(u - u_0) + (-\Delta)^{\beta/2}\, u & = |v|^p, \quad \text{in } Q \\[2mm] D_{0|t}^{\delta}(v - v_0) + (-\Delta)^{\gamma/2}\, v & = |u|^q, \quad \text{in } Q \end{cases}$$

subject to the initial conditions

$$u(x,0) = u_0(x) \geq 0, \quad v(x,0) = v_0(x) \geq 0, \; x \in \mathbb{R}^N,$$

where $0 < \alpha, \delta < 1 \leq \gamma, \beta \leq 2$.

For simplicity, in system **(FDS)** the reaction terms are taken equal to $|v|^p$ and $|u|^q$. Our analysis holds good for reaction terms of the form $f(t,x)|v|^p$ and $g(t,x)|u|^q$ where the functions f and g are assumed to satisfy the conditions

$$f(t,x) \geq C_1 t^{\omega_1}|x|^{d_1}, \qquad g(t,x) \geq C_2 t^{\omega_2}|x|^{d_2},$$

for $t > 0, x \gg 1, \omega_1 \geq 0, \omega_2 \geq 0, d_1 \geq 0, d_2 \geq 0$.

For the system **(FDS)**, we have the

Theorem 2. *Let $p > 1$, $q > 1$. Assume that*

$$N \leq \max\left\{ \frac{\frac{\delta}{q} + \alpha - \left(1 - \frac{1}{pq}\right)}{\frac{\delta}{\gamma q p'} + \frac{\alpha}{\beta q'}}, \frac{\frac{\alpha}{p} + \delta - \left(1 - \frac{1}{pq}\right)}{\frac{\alpha}{\beta p q'} + \frac{\delta}{\gamma p'}} \right\}.$$

*Then, the system **(FDS)** (with the initial data) does not admit nontrivial global weak nonnegative solutions.*

Proof. Here again the proof proceeds by contradiction. For, let

$$\xi_j(x,t) = \Phi\left(\frac{t^2 + |x|^{2\theta_j}}{R^2}\right), \quad j = 1, 2$$

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where $R > 0, \theta_1 = \beta/\alpha$ and $\theta_2 = \gamma/\delta$.

The weak formulation of solutions to system **(FDS)** reads

$$\int_{Q_{TR}} |v|^p \xi_1 + \int_{Q_{TR}} u_0(x) D^\alpha_{t|TR} \xi_1 = \int_{Q_{TR}} u \, D^\alpha_{t|TR} \xi_1 + \int_{Q_{TR}} u(-\Delta)^{\beta/2} \xi_1$$

and

$$\int_{Q_{TR}} |u|^q \xi_2 + \int_{Q_{TR}} v_0(x) D^\delta_{t|TR} \xi_2 = \int_{Q_{TR}} v D^\delta_{t|TR^2} \xi_2 + \int_{Q_{TR}} v \, (-\Delta)^{\gamma/2} \xi_2 \, .$$

Using the Hölder inequality, we may write

$$\int_{Q_{TR}} u \, |D^\alpha_{t|TR^2} \xi_1| \leq \left(\int_{Q_{TR}} |u|^q \xi_2 \right)^{1/q} \cdot \left(\int_{Q_{TR}} | D^\alpha_{t|TR} \xi_1|^{q'} \xi_2^{-\frac{q'}{q}} \right)^{1/q'}$$

and

$$\int_{Q_{TR}} u \, |(-\Delta)^{\beta/2} \xi_1| \leq \left(\int_{Q_{TR}} |u|^q \, \xi_2 \right)^{1/q} \cdot \left(\int_{Q_{TR}} |(-\Delta)^{\beta/2} \xi_1|^{q'} \xi_2^{-q'/q} \right)^{1/q'} ;$$

consequently

$$\int_{Q_{TR}} |v|^p \xi_1 \leq \left(\int_{Q_{TR}} |u|^q \xi_2 \right)^{1/q} \cdot \mathcal{A} \tag{9}$$

with

$$\mathcal{A} = \left(\int_{Q_{TR}} | D^\alpha_{t|TR} \xi_1|^{q'} \xi_2^{-q'/q} \right)^{1/q'} + \left(\int_{Q_{TR}} |(-\Delta)^{\beta/2} \xi_1|^{q'} \xi_2^{-q'/q} \right)^{1/q'} .$$

Similarly, we obtain the estimate

$$\int_{Q_{TR}} |u|^q \xi_2 \leq \left(\int_{Q_{TR}} |v|^p \xi_1 \right)^{1/p} \cdot \mathcal{B} \tag{10}$$

with

$$\mathcal{B} := \left(\int_{Q_{TR}} | D^\delta_{t|TR} \xi_2|^{p'} \xi_1^{-p'/p} \right)^{1/p'} + \left(\int_{Q_{TR}} |(-\Delta)^{\gamma/2} \xi_2|^{p'} \xi_1^{-p'/p} \right)^{1/p'} .$$

Using inequalities (9) and (10), we may write

$$\left(\int_{Q_{TR}} |v|^p \xi_1 \right)^{1 - \frac{1}{pq}} \leq \mathcal{B}^{1/q} \cdot \mathcal{A} \tag{11}$$

and

$$\left(\int_{Q_{TR}} |u|^q \xi_2 \right)^{1 - \frac{1}{pq}} \leq \mathcal{B} \cdot \mathcal{A}^{1/p}. \tag{12}$$

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Now, in $\mathcal{A}$, we use the variables (τ, y) defined by

$$t = R\tau \quad \text{and} \quad x = R^{\frac{\alpha}{\beta}} y,$$

while in $\mathcal{B}$, we use the variables (τ, y) defined by

$$t = R\tau \quad \text{and} \quad x = R^{\frac{\delta}{\gamma}} y.$$

We then have the estimate:

$$\left(\int_{Q_{TR}} |v|^p \xi_1 \right)^{1 - \frac{1}{pq}} \leq C \left\{ R^{-l_1} \right\}^{1/q} R^{-l_2} \tag{13}$$

where

$$l_1 = \delta - \frac{1}{p'} \left(N \frac{\delta}{\gamma} + 1 \right),$$

$$l_2 = \alpha - \frac{1}{q'} \left(N \frac{\alpha}{\beta} + 1 \right).$$

That is,

$$\left(\int_{Q_{TR}} |v|^p \xi_1 \right)^{1 - \frac{1}{pq}} \leq C R^{-\left(\frac{l_1}{q} + l_2 \right)}. \tag{14}$$

Next, we argue as in the case of a single equation (see the argument below formula (5) till the end of the proof) in case $l_1/q + l_2 \geq 0$. Note that the requirement $l_1/q + l_2 \geq 0$ is equivalent to

$$N \leq \frac{\frac{\delta}{q} + \alpha - \left(1 - \frac{1}{pq} \right)}{\frac{\delta}{\gamma q p'} + \frac{\alpha}{\beta q'}}. \tag{15}$$

Using (12), we obtain, in a similar manner, the estimate

$$N \leq \frac{\frac{\alpha}{p} + \delta - \left(1 - \frac{1}{pq} \right)}{\frac{\alpha}{\beta p q'} + \frac{\delta}{\gamma p'}}. \tag{16}$$

Observe that either (15) or (16) is needed to obtain a contradiction, so it suffices to assume

$$1 \leq N \leq \max \left\{ \frac{\frac{\delta}{q} + \alpha - \left(1 - \frac{1}{pq} \right)}{\frac{\delta}{\gamma q p'} + \frac{\alpha}{\beta q'}}, \frac{\frac{\alpha}{p} + \delta - \left(1 - \frac{1}{pq} \right)}{\frac{\alpha}{\beta p q'} + \frac{\delta}{\gamma p'}} \right\}.$$

The case where f and g satisfy the above hypotheses may be proved easily along the lines above and the case of a single equation as in the proof of Theorem 1.

Remark 3. *When $\alpha = \delta = 1, \beta = \gamma = 2$, we recover the case studied by Escobedo and Herrero [7], however we have to impose the constrain $p > 1, q > 1$ while Escobedo and Herrero require $pq > 1$.*

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Remark 4. *It is clear that the more general system*

$$
\begin{cases}
D_{0|t}^{\alpha}(u - u_0) + (-\Delta)^{\beta/2}\left(|u|^{m-1}u\right) & = h(x,t)|v|^p + g(x,t)|u|^r, \qquad in\ Q \\[2mm]
D_{0|t}^{\delta}(v - v_0) + (-\Delta)^{\gamma/2}\left(|v|^{m-1}v\right) & = k(x,t)|u|^q + l(x,t)|v|^s, \qquad in\ Q
\end{cases}
$$

could be analyzed with the same method.

The analysis, here performed, can be used to study systems of convective equations as those, for example, considered by Ames and Straughan [1]. Here, we preferred less general situations to render the ideas as clear as possible.

3   Necessary conditions for local and global existence

This part is concerned with the establishment of necessary conditions for the existence of local (as well as global) solutions to Problems **(STFE)** and **(FDS)**. It turns out that these conditions depend on the behavior of the initial data and on the function $h(x,t)$ ($f(x,t)$ and $g(x,t)$ in case of **(FDS)**) for large x.
Previous results concerning the problem

$$
\begin{cases}
u_t = \Delta u + \widetilde{h}(x)|u|^p, & in\ Q \\
u(x,0) = u_0(x) \geq 0, & in\ \mathbb{R}^N
\end{cases}
\tag{17}
$$

are due to Kalashnikov [12] and to Baras and Kersner [3]. In particular, it is showed in [3] that no local weak nonnegative solution to (17) exists if the initial data u_0 satisfies

$$
\lim_{|x|\to\infty} u_0^{p-1}\widetilde{h}(x) = +\infty,
$$

and any possible local weak nonnegative solution blows-up at a finite time if

$$
\lim_{|x|\to\infty} u_0^{p-1}\widetilde{h}(x)|x|^2 = +\infty.
$$

The method developed there is adapted below to the problem **(STFE)** with, for simplicity $h(x,t) \equiv h(x)$; it will be clear that it can be used for the reaction-diffusion system **(FDS)**.

We shall treat the case of a single equation.

Theorem 3. *Let u be a local solution to Problem **(SFTE)** where $T < +\infty$. Then we have the estimate*

$$
\liminf_{|x|\to\infty} \left[u_0(x)\,(h(x))^{p'/p} \right] \leq C\,T^{\alpha(1-p')}
$$

for some positive constant C.

Proof. Let us consider the following test function

$$\varphi(x,t) = \Phi\left(\frac{x}{R}\right) \begin{cases} \left(1 - \frac{t}{T}\right)^l, & 0 < t \le T \\ 0, & t > T \end{cases}$$

where $\Phi \in W^{1,\infty}\left(\mathbb{R}^N\right)$ is nonnegative with $supp\,\Phi \subset \{1 < |x| < 2\}$ ($supp$ stands for support) and satisfy

$$\left((-\Delta)^{\beta/2}\Phi\right)_+ \le k\Phi, \quad \text{for some constant } k > 0.$$

The exponent l is any positive real number if $p \ge 1/(1-\alpha)$ and $l > \alpha p' - 1$ if $p < 1/(1-\alpha)$. We have

$$D^\alpha_{t|T}\left(1 - \frac{t}{T}\right)^l = \Lambda T^{-\alpha}\left(1 - \frac{t}{T}\right)^{l-\alpha}$$

where $\Lambda := \Gamma(1+l)/\Gamma(1+l-\alpha)$.

Using the formulation (2) and a similar argument to the one which lead us to (4) but keeping the first term in the left hand side of (2), we obtain

$$\int_{Q_T} u_0\, D^\alpha_{t|T}\varphi(x,t) \le C \int_{Q_T} \left\{ \left(D^\alpha_{t|T}\varphi\right)^{p'}_+ + \left((-\Delta)^{\beta/2}\varphi\right)^{p'}_+ \right\} (h\,\varphi)^{1-p'} \tag{18}$$

for some positive constant C. Taking into account the hypotheses on l and the fact that

$$D^\alpha_{t|T}\varphi(x,t) = \Lambda\,\Phi(x)T^{-\alpha}\left(1 - \frac{t}{T}\right)^{l-\alpha},$$

if we put $t = T\tau$ and $x = Ry$ in (18), we obtain

$$T^{1-\alpha}\int_{\mathbb{R}^N} u_0\,(Ry)\Phi(y)$$
$$\le C\,T^{1-\alpha p'}\int_{\mathbb{R}^N} \Phi(y)\,h^{1-p'}(Ry) + C\,T\,R^{-\beta p'}\int_{\mathbb{R}^N} \Phi(y)\,h^{1-p'}(Ry). \tag{19}$$

Using the estimate

$$\inf_{|y|>1}\left(u_0(Ry)h(Ry)^{p'-1}\right)\int_{\mathbb{R}^N}\Phi(y)\,h(Ry)^{1-p'} \le \int_{\mathbb{R}^N} u_0(Ry)\,\Phi(y)$$

in inequality (19) and dividing by the term $\int_{\mathbb{R}^N} u_0(Ry)\,\Phi(y)$, we obtain

$$\inf_{|y|>1}\left(u_0(Ry)h(Ry)^{p'-1}\right) \le C\left(T^{-\alpha(p'-1)} + T^\alpha R^{-\beta p'}\right) \tag{20}$$

Passing to the limit as $R \to +\infty$, we get

$$\liminf_{|x|\to\infty}\left(u_0(x)\,h(x)^{p'-1}\right) \le CT^{-\alpha(p'-1)}. \tag{21}$$

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Corollary 1. *Assume that problem* **(SFTE)** *has a nontrivial global nonnegative weak solution. Then*
$$\liminf_{|x|\to\infty}\left(u_0(x)\,h(x)^{p'-1}\right)=0$$

Corollary 2. *If* $\liminf\limits_{|x|\to\infty}\left(u_0(x)\,h(x)^{p'-1}\right)=+\infty$, *then problem* **(SFTE)** *cannot have any local nontrivial nonnegative weak solution.*

Corollary 3. *If* $A:=\liminf\limits_{|x|\to\infty}\left(u_0(x)\,h(x)^{p'-1}\right)>0$, *then* $T^{\alpha(p'-1)}\le C/A$ *where* C *is the constant found in Theorem 3.*

Theorem 4. *Suppose that problem* **(SFTE)** *has a nontrivial global nonnegative weak solution. Then, there is a positive constant* K *such that*
$$\liminf_{|x|\to\infty}\left(u_0(x)\,|x|^{\alpha(p'-1)}\,h(x)^{1-p'}\right)\le K.$$

Proof. In the relation
$$T^{1-\alpha}\int_{\mathbb{R}^N}u_0\,(Ry)\Phi(y)$$
$$\le C\,(T^{-\alpha(p'-1)}+T^\alpha\,R^{-\beta p'})\int_{\mathbb{R}^N}\Phi(y)\,h^{1-p'}(Ry)$$

found in the proof of Theorem 1, we multiply by the expression
$$h^{p'-1}(Ry)\,|Ry|^{\alpha(p'-1)}.h^{1-p'}(Ry)\,|Ry|^{\alpha(1-p')}$$

inside the integral in the left hand side and by $|Ry|^{\alpha(p'-1)}.h^{1-p'}(Ry)$ inside the integral in the right hand side. We obtain for Φ with $supp\Phi\subset\{x:R<|x|<2R\}$,

$$\inf_{|x|>R}\left(u_0(x)\,|x|^{\alpha(p'-1)}\,h(x)^{p'-1}\right)\int_{\mathbb{R}^N}\Phi(y)\,|Ry|^{\alpha(1-p')}\,h^{1-p'}(Ry)$$
$$\le C\,(T^{-\alpha(p'-1)}+T^\alpha\,R^{-\beta p'})(2R)^{\alpha(p'-1)}\int_{\mathbb{R}^N}\Phi(y)\,|Ry|^{\alpha(1-p')}\,h^{1-p'}(Ry).$$

Finally, dividing by
$$\int_{\mathbb{R}^N}\Phi(y)\,|Ry|^{\alpha(1-p')}\,h^{1-p'}(Ry)$$
and taking $T=R$, we end up with
$$\inf_{|x|>R}\left(u_0(x)\,|x|^{\alpha(p'-1)}\,h(x)^{p'-1}\right)\le C\left(1+R^{(\alpha-\beta)p'}\right).$$

The conclusion follows by passing to the limit and noticing that $\alpha<\beta$.

Combining the argument in the proof of Theorem 2 with those in the previous two theorems, we obtain similar results (necessary conditions for local existence and for global existence) as those in the previous two theorems and their corollaries for the case of system **(FDS)**. The details are omitted.

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References

[1] K. A. Ames and B. Straughan, Boundedness and global nonexistence for a nonlinear convective parabolic system, J. Math. Anal. Appl. 190 (1995), 795–805.

[2] P. Baras and M. Pierre, Critère d'existence de solutions positives pour des équations semi-linéaires non monotones, Ann. Inst. H. Poincaré, Anal. Non Linéaire, 2 (1985), 185–212.

[3] P. Baras and R. Kersner, Local and global solvability of a class of semilinear parabolic equations, J. Diff. Eqs., 68 (1987), 238–252.

[4] P. Biler, T. Funaki and W. A. Woyczynski, Fractal Burgers Equations, J. Diff. Eqs., 148 (1998), 9–46.

[5] W. D. Deering and B. J. West, Fractal physiology, IEEE Engineering in Medicine and Biology, June, (1992) 40-46.

[6] K. Deng and H. A. Levine, The role of critical exponents in blow-up theorems: the sequel, J. Math. Anal. Appl. 243 (2000), 85–126.

[7] M. Escobedo and M. A. Herrero, Boundedness and blow-up for a semilinear reaction-diffusion equation, J. Diff. Eqs. 89 (1991), 176-202.

[8] H. Fujita, On the blowing up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$, J. Fac. Sci. Univ. Tokyo Sect. I 13 (1966), 109–124.

[9] M. Guedda and M. Kirane, A note on nonexistence of global solutions to a nonlinear integral equation, Bull. Belg. Math. Soc. Simon Stevin 6 (1999), 91–497.

[10] M. Guedda and M. Kirane, Criticality for some evolution equations, Differ. Eqs. 37 (2001), 540–550.

[11] K. Hayakawa, On nonexistence of global solutions of some semilinear parabolic differential equations, Proc. Japan Acad. 49 (1973), 503–505.

[12] A. S. Kalashnikov, On a heat conduction equation for a medium with non-uniformy distributed non-linear heat source or absorbers, Bull. Univ. Moscow Math. Mech. 3 (1983), 20-24.

[13] M. Kirane and M. Qafsaoui, Global nonexistence for the Cauchy problem of some nonlinear reaction-diffusion systems, J. Math. Anal. Appl. 268 (2002), 217–243.

[14] K. Kobayashi, T. Sirao and H. Tanaka, On the growing up problem for semi-linear heat equations, J. Math. Soc. Japan 29 (1977), 407–424.

[15] H. Kuiper, Life span of nonnegative solutions to certain quasilinear parabolic Cauchy problems, Electronic . J. of Differential Equations, No 66 (2003), 1-11.

[16] H. A. Levine, Some nonexistence and instability theorems for solutions of formally parabolic equations of the form $Pu_t = -Au + F(u)$, Arch. Rational Mech. Anal. 51 (1973), 371–386.

[17] G. G. Laptev, Nonexistence results for higher-order evolution partial differential inequalities, Proc. Amer. Math. Soc. 131 (2003), 415–423 (electronic).

[18] E. Mitidieri and S. I. Pokhozhaev, A priori estimates and blow-up of solutions to nonlinear partial differential equations and inequalities, Proc. Steklov Inst. Math., 234 (2001), 1–383.

[19] E. W. Montroll and G. H. Weiss, Random walks, II, J. Math. Phys. 6 (1965), 167–181.

[20] M. Nagasawa and T. Sirao, Probabilistic treatment of the blowing up of solutions for a nonlinear integral equation, Trans. Amer. Math. Soc. 139 (1969) 301–310.

[21] R. R. Nigmatullin, A fractional integral and its physical interpretation, Theoret. Math. Phys. 90 (1992), 242–251.

[22] L. E. Payne, "Improperly posed problems in partial differential equations", Regional Conference Series in Applied Mathematics, No. 22. Society for Industrial and Applied Mathematics, Philadelphia, Pa., 1975.

[23] I. Podlubny, " Fractional Differential Equations", Math. in Sci. and Eng.: Vol. 198 New-York-London: Acad. Press 1999.

[24] S. I. Pokhozhaev and A. Tesei, A. Critical exponents for the absence of solutions for systems of quasilinear parabolic inequalities, Differ. Eqs. 37 (2001), 551–558

[25] Y.-W. Qi, The critical exponents of parabolic equations and blow-up in R^n, Proc. Roy. Soc. Edinburgh Sect. A 128 (1998), 123–136.

[26] H. E. Roman and M. Giona, Fractional diffusion equation on fractals: three-dimensional case and scattering function, J. Phys. A 25 (1992), 2107–2117.

[27] A. A. Samarski, V. A Galaktionov, S. P. Kurdyumov and A. P. Mikhailov, "Blowups in Problems for Quasilinear Parabolic Equations", de Gruyter Expositions in Mathematics, Vol. 19, de Gruyter, Berlin, 1995.

[28] E. Scalas, R. Gorenflo and F. Mainardi, Fractional calculus and continuous-time finance, Phys. A 284 (2000), 376–384.

[29] M. Seredynska and A. Hanyga, Nonlinear Hamiltonian equations with fractional damping, J. Math. Phys. 41 (2000), 2135–2156.

[30] B. Straughan, "Explosive instabilities in mechanics", Springer-Verlag, Berlin, 1998.

[31] S. Sugitani, On nonexistence of global solutions for some nonlinear integral equations, Osaka J. Math. 12 (1975), 45–51.

[32] S. Q. Zhang , A blow-up result for a nonlinear wave equation with damping: the critical case, C. R. Acad. Sci. Paris, 333 (2001), 109–114.

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